# Advanced (International) Macroeconomics

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Fall 2024

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# 2-Period Model

# 2-period model

**Within periods:**
Inter- and intraindustry trade (see trade theory from Ricardo, Heckscher/Ohlin to new trade theories with imperfect markets). Gains from international labor division (comparative advantages), exploitation of economies of scale or intensified competition.

**Across periods:**
Intertemporal trade. Gains from international borrowing and lending.

# Intertemporal choice - preferences

$$U = u(c_1) + \beta u(c_2), \quad 0 < \beta < 1 \tag{1}$$

- $u$ period (instantaneous) utility
- $u' > 0, u'' < 0, \lim_{c \to 0} u'(c) = \infty$
- $\beta$ subjective discount factor (time-preference parameter)

**Marginal rate of substitution**

$$MRS \left( \equiv -\frac{dc_2}{dc_1}\bigg|_{U=const} \right) = \frac{u'(c_1)}{\beta u'(c_2)} \tag{2}$$

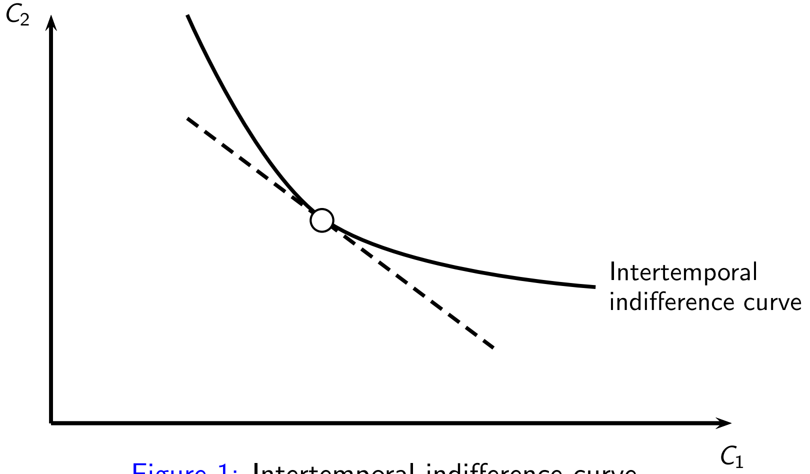



Figure 1: Intertemporal indifference curve

# Utility functions and intertemporal budget constraint

**Isoelastic utility functions**

- Example 1: $u(c_t) = \ln c_t$
- Example 2: $u(c_t) = \dfrac{c_t^{1-1/\sigma}}{1-1/\sigma} \quad , \qquad 0 < \sigma (\neq 1)$.

**Intertemporal budget constraint**

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} \tag{3}$$

- $r$ interest rate (if there are variable interest rates, $r_{t+1}$ denotes interest rate from $t$ to $t+1$ . Then $\dfrac{c_2}{1+r_2}$ etc.).
- $y_t$ endowment (income) in period $t$.

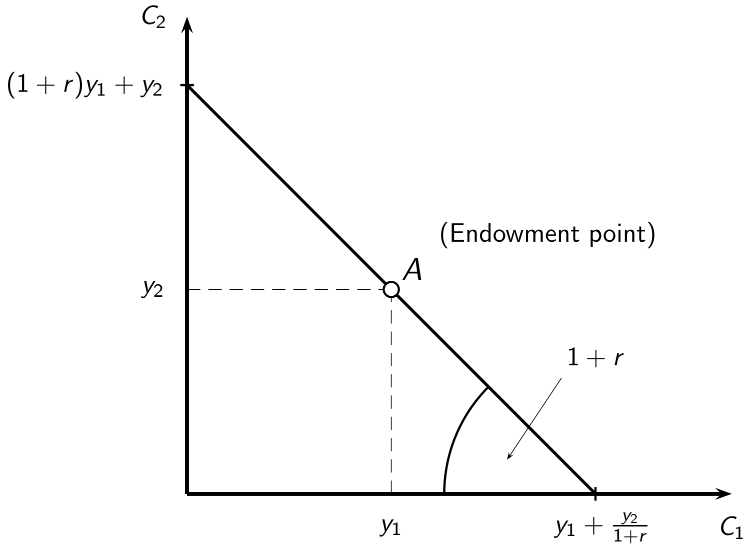



Figure 2: Intertemporal budget constraint

# Optimal intertemporal choice

$$\max \quad U \qquad \text{s.t.} \quad (3)$$

Lagrange-Function:

$$\mathcal{L} = u(c_1) + \beta u(c_2) + \lambda \left( y_1 + \frac{y_2}{1+r} - c_1 - \frac{c_2}{1+r} \right)$$

# Optimal intertemporal choice

First-order conditions

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_1} &= 0 \implies u'(c_1) = \lambda \\
\frac{\partial \mathcal{L}}{\partial c_2} &= 0 \implies \beta u'(c_2) = \frac{\lambda}{1+r} \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= 0 \implies (3)
\end{aligned}$$

combine to the so-called *intertemporal Euler equation*:

$$u'(c_1) = (1+r)\beta u'(c_2) \tag{4}$$

Eq. (4) determines how consumption needs to be allocated intertemporally in order to maximize utility at a given interest rate $r$.

# Interpretation of Euler Equation

The intertemporal choice is optimal if there are no gains from intertemporal reallocation. Equation (4) is equivalent to

$$MRS = 1 + r \tag{5}$$

where $MRS$ is given by (2)

# Equilibrium in an endowment economy

# Equilibrium in an endowment economy

Endowment economies have no capital accumulation and no production.

**Aggregate supply (with symmetric agents):**
$Y_t = y_t N_t, \ t = 1, 2$ where $N_t$ is the population size in period $t$. In the following, $N_t$ is normalized to 1 so that $Y_t = y_t$.

**Aggregate demand:**
$C_t = c_t, \ t = 1, 2$

# Equilibrium in an endowment economy

In **closed economy** (autarky):

$$C_t = Y_t, \; t = 1, 2 \tag{6}$$

In **open economy**:

$$C_t = Y_t + r_t B_t - CA_t \tag{7}$$

where $B_t$ is the value of *net foreign assets* inherited from period $t-1$ and $CA_t$ is the *current account balance* (Ertragsbilanz, auch Leistungsbilanz).

By definition,

$$CA_t = B_{t+1} - B_t \tag{8}$$

# Remarks on national accounting

- *Gross domestic product* (GDP) (Bruttoinlandsprodukt): $Y_t$
- *Gross national product* (GNP) (Bruttosozialprodukt): $Y_t + r_t B_t$ i.e.
  - GNP=GDP+net international factor payments
  - net international factor payments here only includes interest and dividend earnings on net foreign assets
  - but no workers remittances
- *Trade balance* (goods and services): Net exports $NX_t$

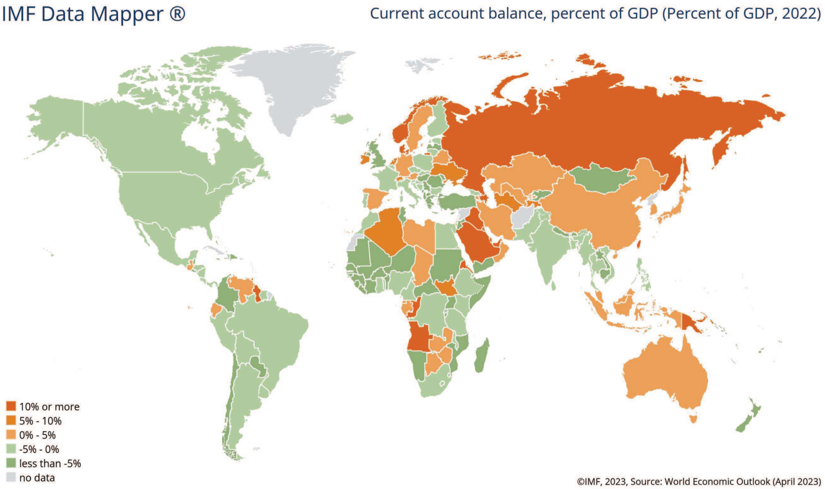
# Remarks on national accounting

- *Capital account balance* (Kapitalverkehrsbilanz, auch Kapitalbilanz; includes financial account balance): Net sales of foreign assets: $-(B_{t+1} - B_t)$
- *Balance of payments* (Zahlungsbilanz): $NX_t + r_t B_t = B_{t+1} - B_t$
- *Current account balance* (Ertragsbilanz, auch Leistungsbilanz):

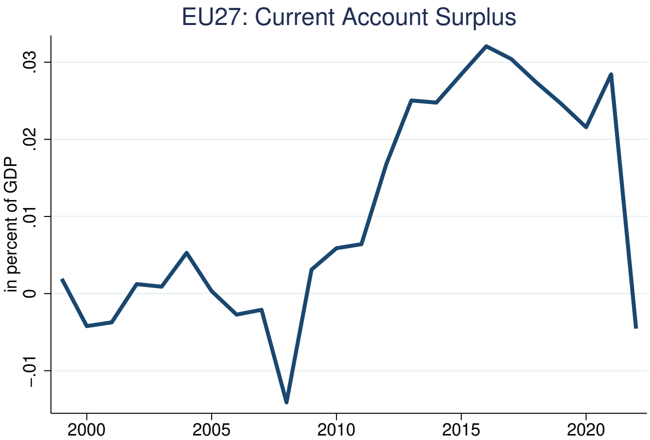
$$\text{within period perspective: } CA_t = NX_t + r_t B_t \tag{9}$$

$$\text{intertemporal perspective: } CA_t = B_{t+1} - B_t \tag{10}$$

# Remarks on national accounting

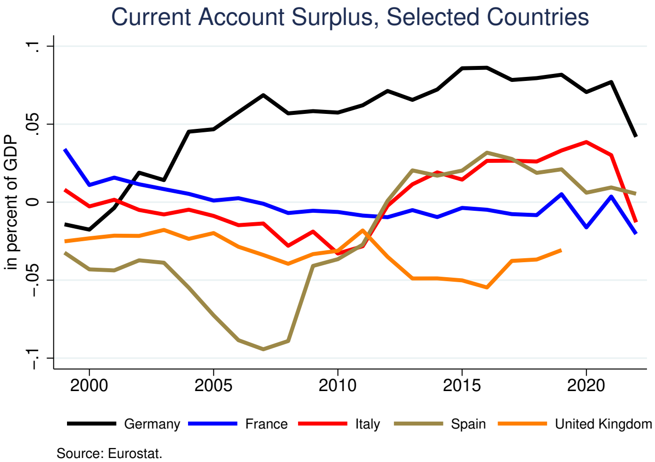


# Remarks on national accounting

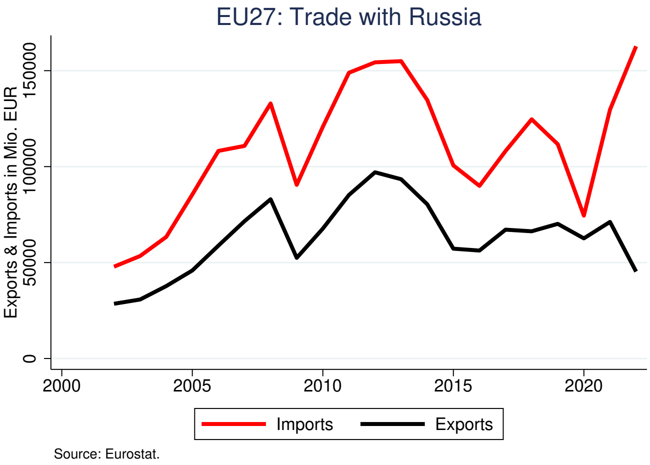


Source: Eurostat.

# Remarks on national accounting



# Remarks on national accounting



# Remarks on national accounting

Major items of the balance of payments (€ billions)

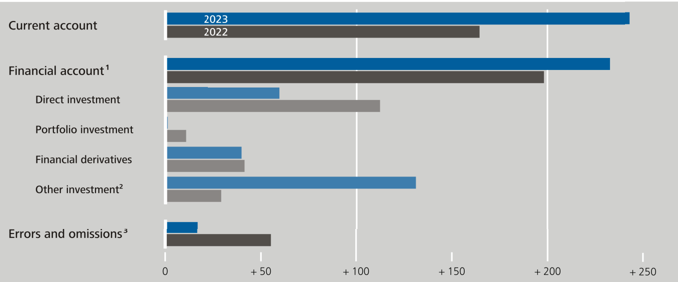
| Item | 2021 | 2022 | 2023 |
|---|---|---|---|
| *I. Current account* | | | |
| Total | +163.5 | +164.6 | +243.1 |
| 1. Goods[1] | +196.5 | +125.9 | +226.8 |
| 2. Services | +1.6 | –37.3 | –63.8 |
| 3. Primary income | +123.2 | +142.1 | +143.9 |
| 4. Secondary income | –57.8 | –66.1 | –64.6 |
| *II. Capital account* | | | |
| Total | -2.6 | -21.6 | -27.3 |
| *III. Financial account balance*[2] | | | |
| Total | +209.0 | +198.2 | +232.6 |
| 1. Direct investment | +81.5 | +112.2 | +59.7 |
| 2. Portfolio investment | +197.2 | +10.7 | +1.2 |
| 3. Financial derivatives[3] | +47.9 | +41.5 | +40.0 |
| 4. Other investment[4] | –149.5 | +29.3 | +130.9 |
| 5. Reserve assets | +31.9 | +4.4 | +0.9 |
| *IV. Errors and omissions* | | | |
| Total | –51.9 | +55.2 | +16.7 |

Source: Deutsche Bundesbank: Monthly Report, March 2024. [1] Mainly foreign trade. [2] Increase in net external position: + / decrease in net external position: -. [3] Arising from options and financial futures contracts or employee stock options. [4] Includes loans and trade credits as well as currency and deposits.

# Remarks on national accounting

**Major items of the German balance of payments** Chart 4.8

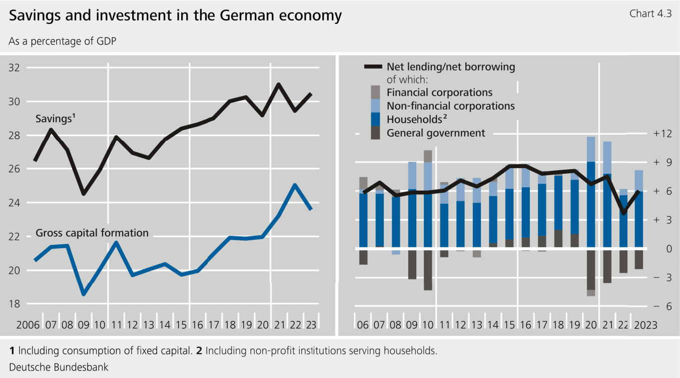
Balances in € billion




**1** Net capital exports: +. **2** Includes, in particular, loans and trade credits as well as currency and deposits. **3** Statistical errors and omissions.
Deutsche Bundesbank

Source: Deutsche Bundesbank: Monthly Reports, March 2024

# Remarks on national accounting




Source: Deutsche Bundesbank: Monthly Reports, March 2024

# Remarks on national accounting

Long-run impact of short-run imbalances:

$$CA_t = NX_t + r_t B_t \text{ and } CA_t = B_{t+1} - B_t$$

imply

$$B_{t+1} = NX_t + (1 + r_t)B_t$$

Repeating the argument for $B_{t+2}$ we get

$$B_{t+2} = NX_{t+1} + (1 + r_{t+1})NX_t + (1 + r_t)(1 + r_{t+1})B_t$$

# Remarks on national accounting

In a $T+1$-period world with $r_t = r$ (inheriting $B_t$ and leaving $B_{t+T+1}$):

$$\begin{aligned} B_{t+T+1} &= (1+r)^{T+1} B_t + (1+r)^T NX_t + \dots \\ &\quad + (1+r) NX_{t+T-1} + NX_{t+T} \end{aligned}$$

$$\iff$$

$$\left(\frac{1}{1+r}\right)^T B_{t+T+1} = (1+r) B_t + \sum_{s=t}^{t+T} \left(\frac{1}{1+r}\right)^{s-t} NX_s \quad (11)$$

# Remarks on national accounting

Terminal condition

$$B_{t+T+1} = 0$$

implies

$$\sum_{s=t}^{t+T} \left( \frac{1}{1+r} \right)^{s-t} NX_s = -(1+r)B_t$$

# Remarks on national accounting

For instance, for $B_3 = B_1 = 0$ (temporary imbalance[1]):

$$\begin{aligned} NX_1 + \frac{NX_2}{1+r} &= -(1+r)B_1 = 0 \\ CA_1 + CA_2 &= 0 \end{aligned}$$

and

$$\begin{aligned} CA_1 &= B_2 - B_1 = B_2 \\ CA_2 &= B_3 - B_2 = -B_2 \\ NX_1 &= CA_1 - rB_1 = CA_1 = B_2 \\ NX_2 &= CA_2 - rB_2 = -(1+r)B_2 \end{aligned}$$

[1] if interested in more long-run dynamics, see Obstfeld/Rogoff Chapter 2

# Equilibrium in a closed economy

# Equilibrium in a closed economy

Goods market equilibrium

$$C_t = Y_t,\ t = 1, 2$$

and optimal consumption choice (cf. intertemporal Euler equation)

$$u'(C_1) = (1 + r)\beta u'(C_2)$$

give us *the autarky real interest rate*

$$1 + r^A = \frac{u'(Y_1)}{\beta u'(Y_2)} \tag{12}$$

# Equilibrium in a closed economy

(Budget constraint (3) is obviously fulfilled for $C_t = Y_t$.)

- $1 + r^A$ is the willingness to pay for present consumption.
- Virtual price in closed economy without investment possibilities.
- Relevant when opening up.

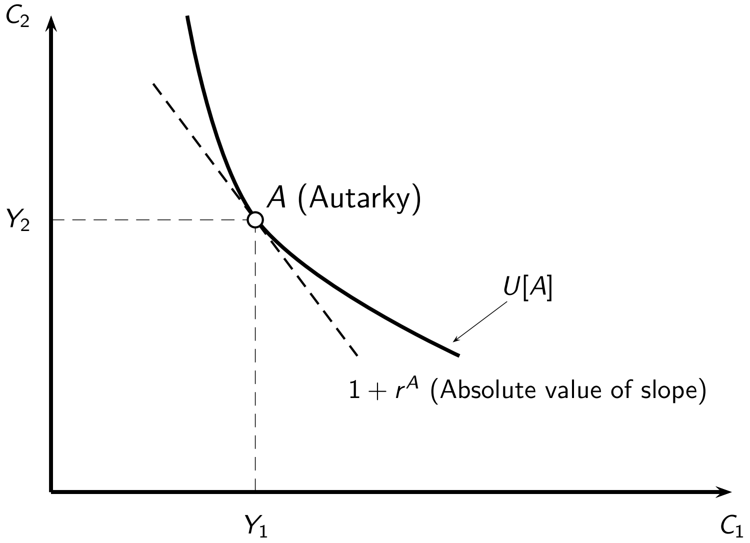



Figure 3: Autarky equilibrium

# Effect of time preference on $r^A$

$\beta > \widetilde{\beta}$ implies

$$MRS(Y_1, Y_2) \quad < \quad \widetilde{MRS}(Y_1, Y_2)$$

$$\text{where } MRS(Y_1, Y_2) \quad \equiv \quad \frac{u'(Y_1)}{\beta u'(Y_2)}$$

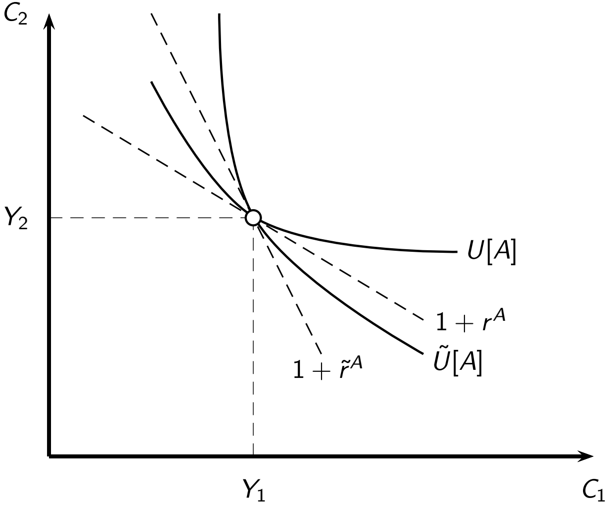



Figure 4: $r^A$ rises with impatience.

# Effect of output changes on $r^A$

Assume linear consumption expansion path
$(MRS(\lambda Y_1, \lambda Y_2) = MRS(Y_1, Y_2))$

- $r^A$ rises if positive output shock is expected.
- No change of $r^A$ if present and future output rise pari passu.

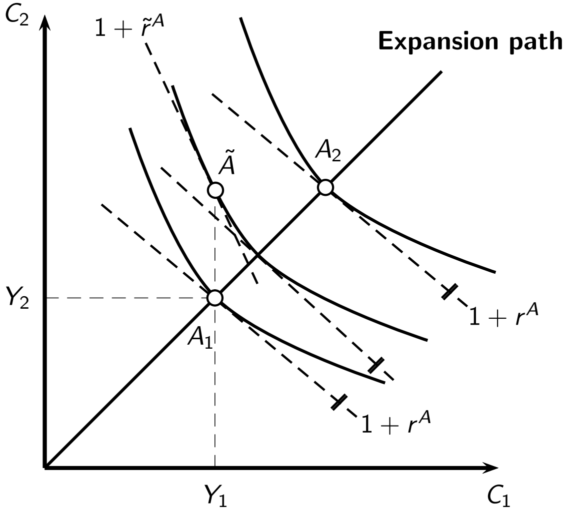



Figure 5: Effect of output changes on $r^A$

# Equilibrium in a small open economy

# Equilibrium in a small open economy

- 2 periods: $B_1 = B_3 = 0$ i.e. $NX_1 + \frac{NX_2}{1+r} = 0$
- $r$ exogenously given by the world market

Intertemporal equilibrium allocation $C_1, C_2$ determined by:

$$u'(C_1) = (1+r)\beta u'(C_2) \quad (13)$$

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r} \quad (14)$$

# Special case

If subjective discount factor is equal to market discount factor

$$\beta = \frac{1}{1+r},$$

the solution of (13) & (14) is given by

$$C_1 = C_2 \equiv \overline{C} \tag{15}$$

$$\overline{C} = \frac{(1+r)Y_1 + Y_2}{2+r} \tag{16}$$

For $\beta < \dfrac{1}{1+r}$ the allocation is biased in favor of $C_1$.

# Open vs. closed economy equilibrium

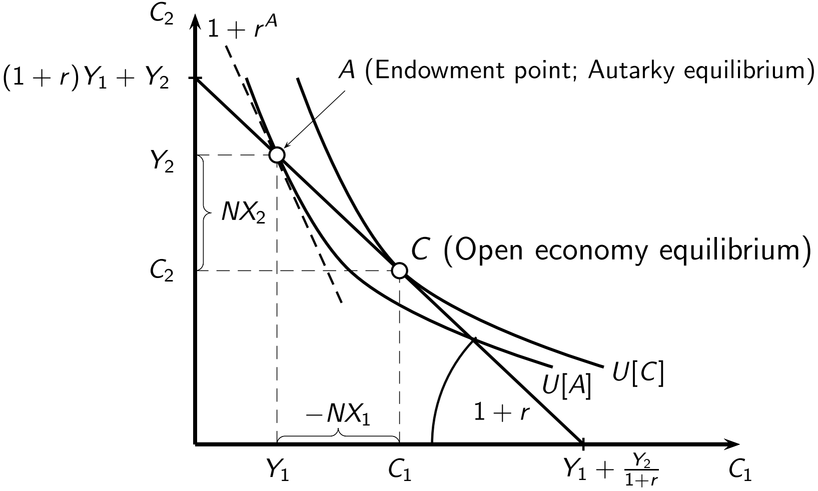



Figure 6: Comparing open and closed equilibrium if $r_A > r$

# Open vs. closed economy equilibrium

- Access to international capital markets allows intertemporal income shifting.
- Here from future to present (borrowing) since $r < r^A$.
- Gains from intertemporal trade $U[C] > U[A]$
- Debt from current net imports $NX_1 = Y_1 - C_1 < 0$ must be paid back by future net exports
  $NX_2 = -(1+r)NX_1 = Y_2 - C_2 > 0$

# Implications for trade flows and capital account

According to (7), (9) and (10):

$$
\begin{aligned}
C_1 = \quad Y_1 + rB_1 - CA_1 &= Y_1 + (1+r)B_1 - B_2 \\
&= Y_1 - NX_1
\end{aligned}
$$

$$
\begin{aligned}
C_2 = \quad Y_2 + rB_2 - CA_2 &= Y_2 + (1+r)B_2 - B_3 \\
&= Y_2 - NX_2
\end{aligned}
$$

# In 2-period world: $B_1 = B_3 = 0$

**Trade flows**

$$\begin{aligned} C_1 - Y_1 &= -NX_1 \\ Y_2 - C_2 &= (1+r)(C_1 - Y_1) = NX_2 \end{aligned}$$

**Net foreign assets**

$$\begin{aligned} B_2 &= -(C_1 - Y_1) \\ Y_2 - C_2 &= -(1+r)B_2 = (1+r)(C_1 - Y_1) \end{aligned}$$

# “Long-run” effects of short-run trade deficit

If endowment expectations are wrong, the associated short run trade deficit may have long-run implications (3 or more periods, $B_1 = 0$).

- $CA_1 = NX_1 = B_2$
- $CA_2 = rB_2 = rNX_1$
- $B_3 = CA_2 + B_2 = (1 + r)NX_1$

$B_{t+1} = B_t,\ t \geqslant 3$ requires $CA_t = NX_t + rB_t = 0$

$$\Longrightarrow \text{For } t \geqslant 3\text{: } NX_t = -r(1 + r)NX_1 \Longrightarrow C_{t(\geqslant 3)} < Y_t$$

$B_4 = 0$ requires $CA_3 = -B_3$

$$\Longrightarrow NX_3 = -(1 + r)^2 NX_1 \Longrightarrow \widetilde{C}_3 < C_{t(\geqslant 3)} < Y_3$$

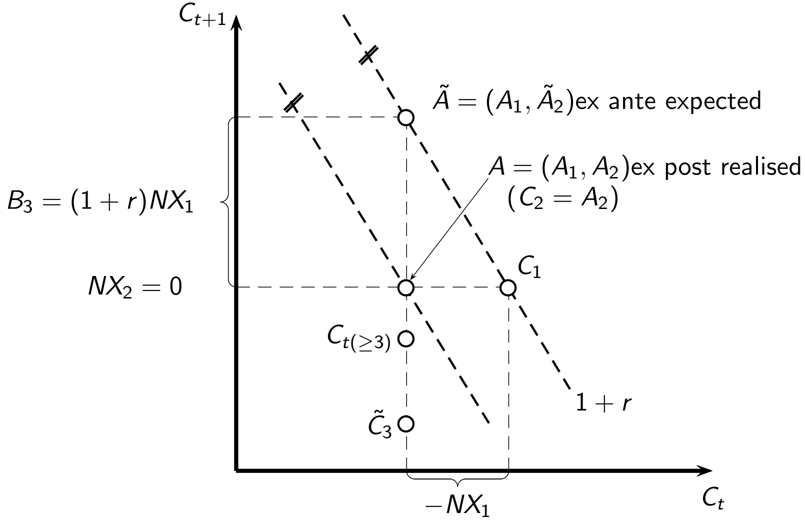



Figure 7: "Long-run" effects of trade deficit (3 per., $B_1 = 0$)

# Intertemporal trade, savings and international equilibrium

# Intertemporal trade, savings and international equilibrium

**Case** $r < r^A$

(see figure 6): Implies $NX_1 < 0,\ NX_2 > 0$, given $B_1 = B_3 = 0$.

- The country is a net importer in period 1, net exporter in period 2.
- $1 + r$ is the price of present consumption (here the import good) in terms of future consumption (export good).

# Intertemporal trade, savings and international equilibrium

**Terms of trade**

$$\text{Terms of trade} = \frac{\text{price of exports}}{\text{price of imports}} = \frac{1}{1+r}$$

*decline* in $r$ $\Rightarrow$ i) terms of trade improve

$\Rightarrow$ positive income and wealth effect on $C_1$

ii) substitution effects also favors $C_1$

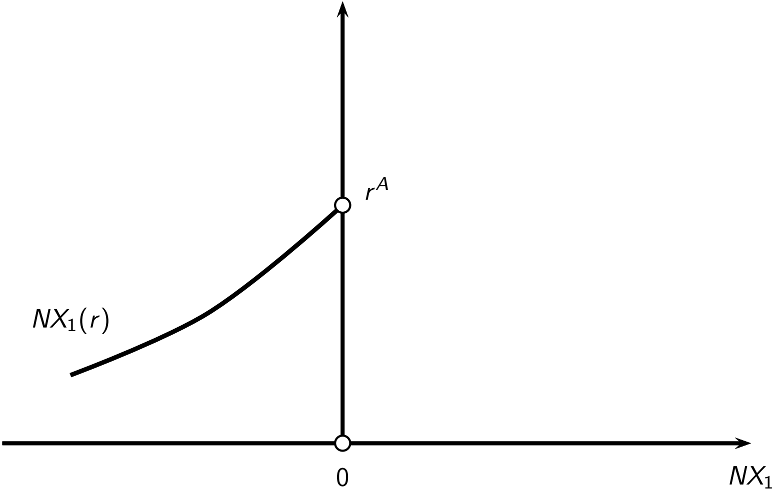



Figure 8: Net exports and interest rate if $r < r^A$

# Intertemporal trade, savings and international equilibrium

**Case** $r > r^A$

(see figure 9): Implies $NX_1 > 0$, $NX_2 < 0$, given $B_1 = B_3 = 0$.

- Country is net exporter of present output, terms of trade $1 + r$

$$\begin{aligned} \textit{rise in } r \quad &\Rightarrow \quad \text{i) positive terms of trade effect on } C_1 \\ &\Rightarrow \quad \text{ii) negative substitution effect on } C_1 \end{aligned}$$

In sum, the $NX_1$-reaction is ambiguous.

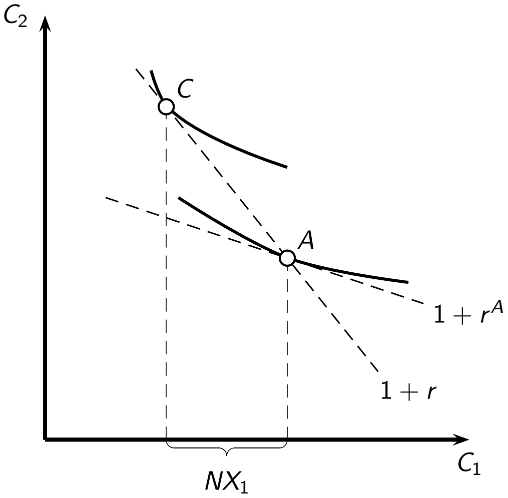



Figure 9: Intertemporal trade with $r > r^A$

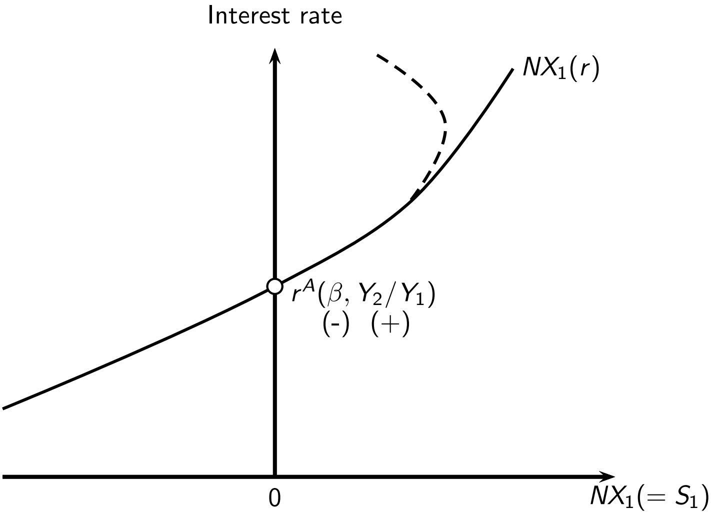



Figure 10: Net exports and interest rate if $r > r^A$

# Intertemporal trade, savings and international equilibrium

The position of the curve is fixed by $r^A$.

Remember: $r^A$ declines in $\beta$ and (for linear expansion path of consumption) rises with $Y_2/Y_1$.

Moreover: For $B_1 = 0$, $CA_1 = NX_1$ and thus
$S_1 \equiv Y_1 + rB_1 - C_1 = NX_1$

# International equilibrium – 2 large economies

Two country world:

$$\text{Home:} \quad r^A(\beta, Y_2/Y_1)$$
$$\text{Foreign:} \quad r^{A^*}(\beta^*, Y_2^*/Y_1^*)$$

integrated world is like closed economy with goods market equilibrium condition

$$C_t + C_t^* = Y_t + Y_t^*$$

Using $C_t + NX_t = Y_t,\ C_t^* + NX_t^* = Y_t^*$, we get

$$NX_t + NX_t^* = 0 \tag{17}$$

This determines world interest rate $r$.

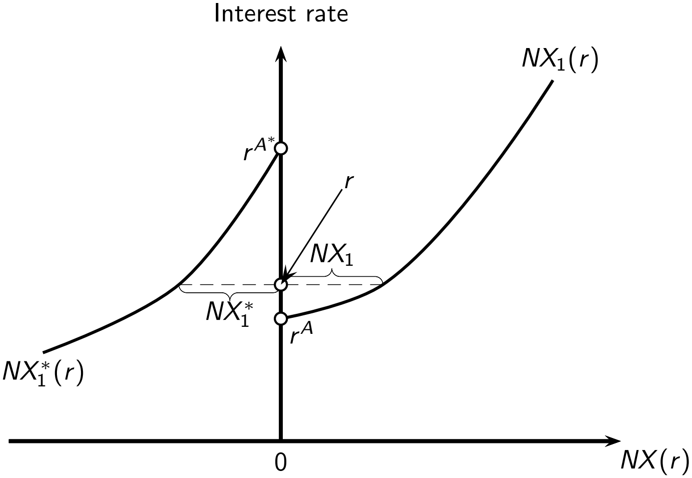



Figure 11: Equilibrium world interest rate if $r^{A^*} > r^A$

# International equilibrium – 2 large economies

If increasing impatience ($\beta$ or $\beta^* \downarrow$) or rising future output ($Y_2/Y_1$ or $Y_2^*/Y_1^* \uparrow$) raise $r^A$ or $r^{A^*}$ the equilibrium world interest rate rises, ceteris paribus. This

- worsens terms of trade for net importer Foreign
- improves terms of trade for net exporter Home

# Capital accumulation and production

# Capital accumulation and production – Assumptions

**Production function**:

$$Y_t = F\left(K_t\right)$$

Properties:

- $F(0) = 0$
- $F' > 0$
- $F'' < 0$

Inada conditions:

- $\lim_{K \to 0} F'(K) = \infty$
- $\lim_{K \to \infty} F'(K) = 0$

Since $N_t = 1$, level of capital stock $K_t$ and capital intensity $k_t = K_t / N_t$ coincide.

# Capital accumulation and production – Assumptions

**Capital accumulation:**

$$K_{t+1} = K_t + I_t \tag{18}$$

(Depreciation ignored, $K$ can be eaten up, i.e. $I_t = -K_t$.)

**Capital demand under perfect competition:**

$$r_t = F'(K_t) \tag{19}$$

**Wages (labor demand) under perfect competition:**

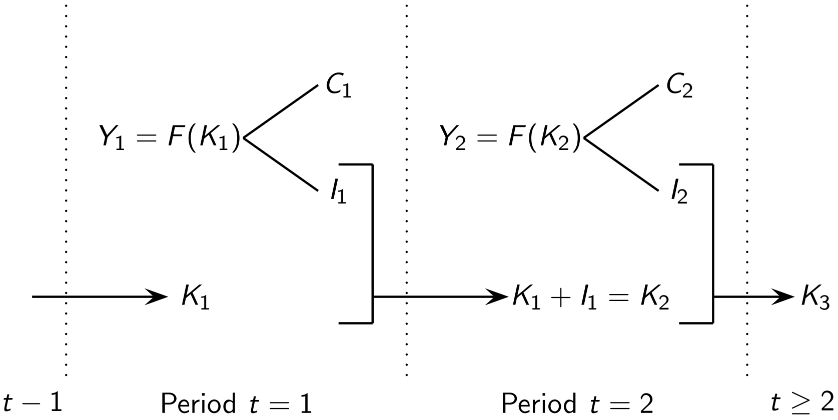
$$W_t = F(K_t) - r_t K_t \tag{20}$$

# Closed economy with capital accumulation and production

# Intertemporal production and investment – Autarky

Goods market equilibrium

$$C_t + I_t = Y_t \tag{21}$$



# Intertemporal transformation curve ("Production possibilities Frontier" PPF)

$$
\begin{aligned}
C_2 + K_3 &= F(K_2) + K_2 \\
&= F\left[K_1 + \underbrace{F(K_1) - C_1}_{I_1}\right] + K_1 + F(K_1) - C_1 \quad (22)
\end{aligned}
$$

**Intertemporal PPF:**

$$C_2^+ \equiv C_2 + K_3$$

$$C_2^+ = F\left(\underbrace{K_1 + F(K_1) - C_1}_{K_2}\right) + \underbrace{K_1 + F(K_1) - C_1}_{K_2}$$

# Intertemporal transformation curve ("Production possibilities Frontier" PPF)

$$\frac{dC_2^+}{dC_1} = -\left[1 + F'\left(\underbrace{K_1 + F(K_1) - C_1}_{K_2}\right)\right] < 0$$

$$\frac{d^2 C_2^+}{dC_1^2} = F''(K_2) < 0$$

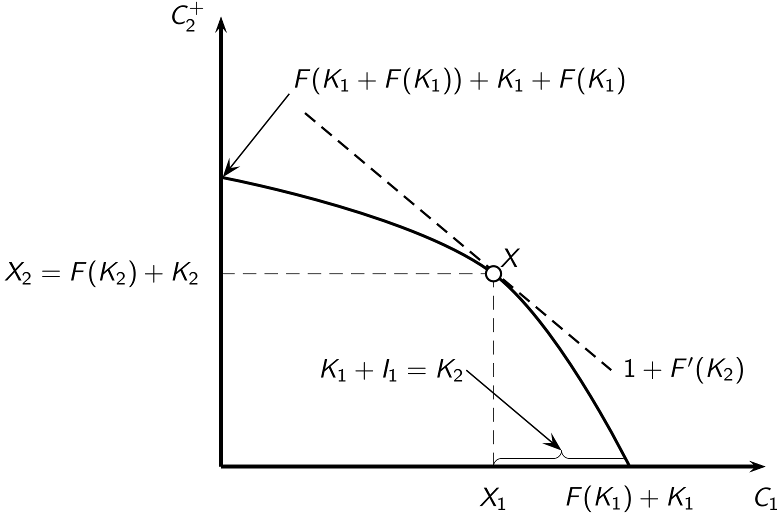
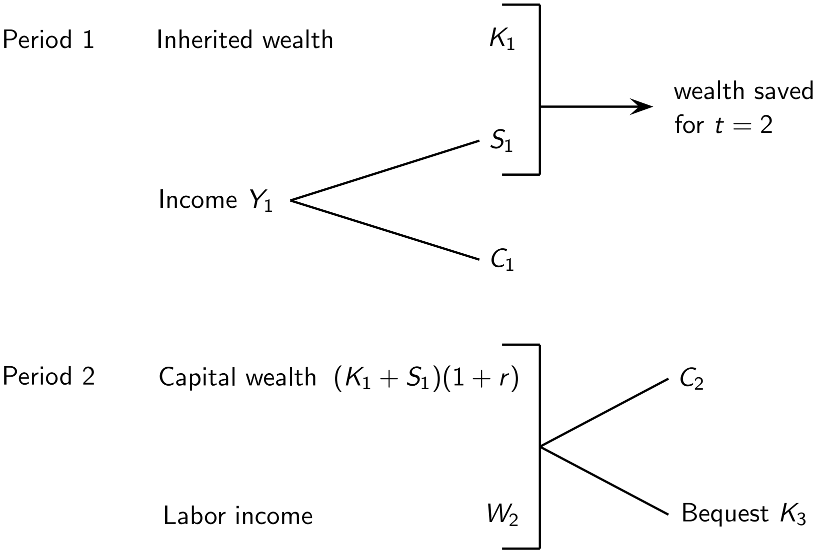



Figure 12: Intertemporal production possibilities frontier

# Intertemporal budget constraint of representative HH



# Intertemporal consumption possibility line (CPL)

Intertemporal budget constraint:

$$C_2 + K_3 = W_2 + (K_1 + Y_1 - C_1)(1 + r) \tag{23}$$

Since $W_2 = F(K_2) - rK_2$ and $K_1 + Y_1 - C_1 = K_2$ (23) is consistent with (22).

That means: Savings behavior of households leads to a point on the economy's PPF.

The question is: which point?

# Optimal intertemporal choice

$K_3$- choice depends on the "bequest" motive. Can be captured by

$$u(C_1) + \beta u(C_2^+)$$

$$\begin{aligned} C_2^+ &= C_2 + K_3 \ldots \text{ ``bequest'' motive} \\ C_2^+ &= C_2 \ldots \text{ no ``bequest'' motive} \end{aligned}$$

$$\max\ u(C_1) + \beta u(C_2^+) \text{ s.t. } C_2^+ = W_2 + (K_1 + Y_1 - C_1)(1 + r)$$

# Optimal intertemporal choice

Optimal intertemporal choice yields first-order condition

$$MRS \equiv \frac{u'(C_1)}{\beta u'(C_2^+)} = 1 + r$$

where $K_3 = 0$ without bequest motive. ($K_3 = 0$ implies $K_2 + I_2 = 0$ and thus $S_2 = I_2 = -K_2$.) In dubio, assume $K_3 = 0$, i.e. $C_2 = C_2^+$ in the following.

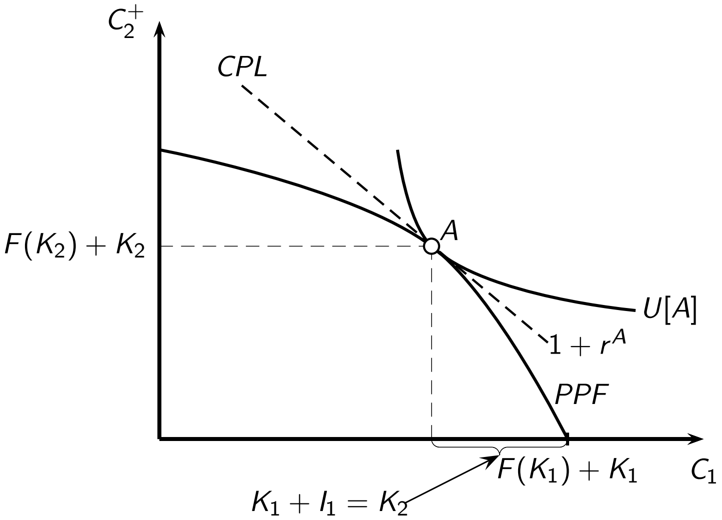



Figure 13: CPL and PPF

# Small open economy with capital accumulation and production

# SMOPEC with capital accumulation and production

Goods market equilibrium

$$C_t + I_t + NX_t = Y_t \tag{24}$$

and intertemporal foreign account (see (11))

$$NX_2 + (1+r)NX_1 = \underbrace{B_3 - (1+r)^2 B_1}_{D}$$

imply

$$\begin{aligned} C_2 + (1+r)C_1 &= Y_2 - I_2 + (1+r)(Y_1 - I_1) - D \\ &= F(K_2) - [K_3 - K_2] + (1+r)[F(K_1) - \\ &\quad (K_2 - K_1)] - D \end{aligned}$$

# SMOPEC with capital accumulation and production

Hence,

$$\underbrace{C_2 + K_3}_{C_2^+} + (1+r)C_1 = \underbrace{F(K_2) + K_2}_{X_2} + (1+r)\underbrace{[F(K_1) + K_1 - K_2]}_{X_1} - D \tag{25}$$

where $X = (X_1, X_2)$ is a point at the PPF.

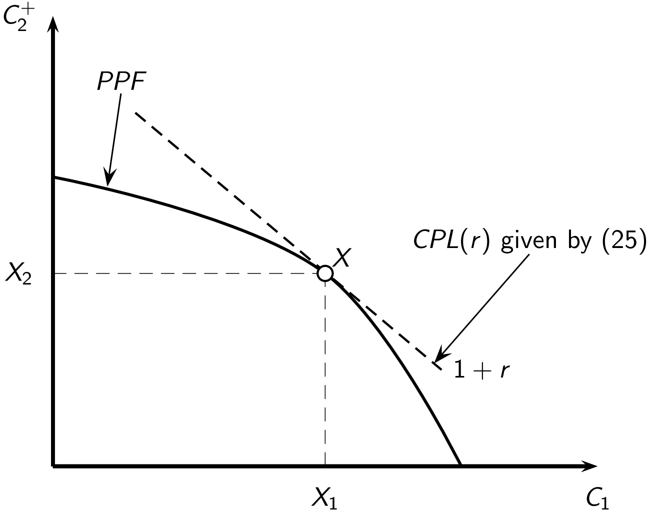



Figure 14: Consumption possibilities line (CPL) under world interest and $D = 0$.

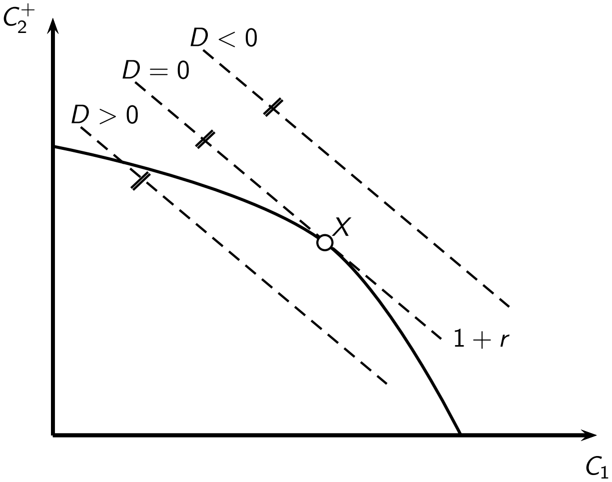



Figure 15: CPL under long-run imbalances ($D \neq 0$)

In the following $D = 0$ (e.g. $B_1 = B_3 = 0$).

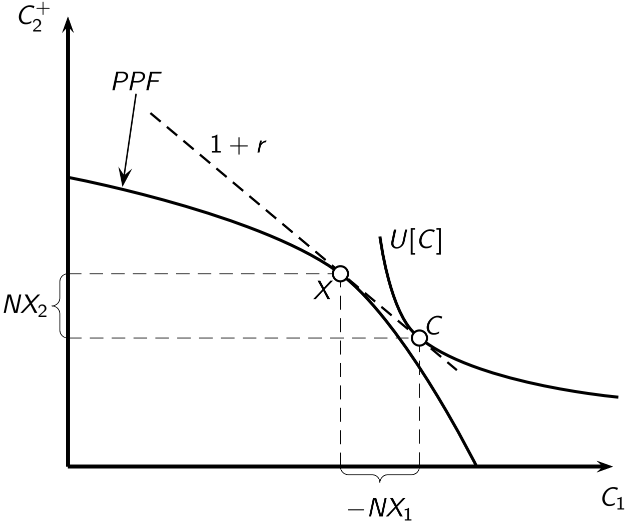



Figure 16: Equilibrium production ($X$) equilibrium consumption ($C$), and trade balances ($NX$)

# From autarky to open economy equilibrium

Consider $r < r^A$:

In addition to the picture for the endowment economy: Production structure shifts from $A$ to $X$ by higher investments $\Delta I_1$.

Increase in current consumption by $\Delta C_1$. Current account deficit $-NX_1 = \Delta I_1 + \Delta C_1$ paid back by increased future production (+ possibly lower consumption).

**Case $r < r^A$:**

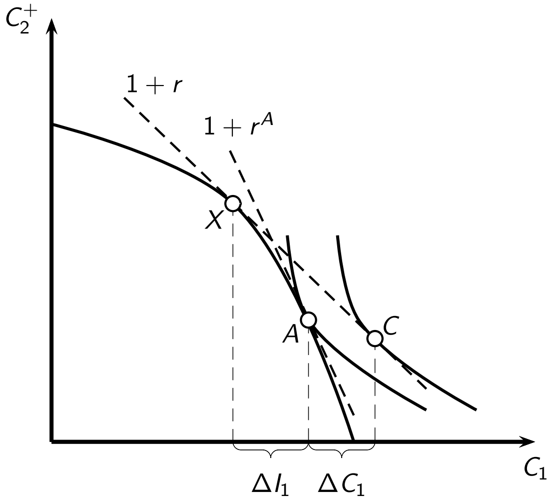



Figure 17: Double gains from intertemporal trade

**Case $r > r^A$:**

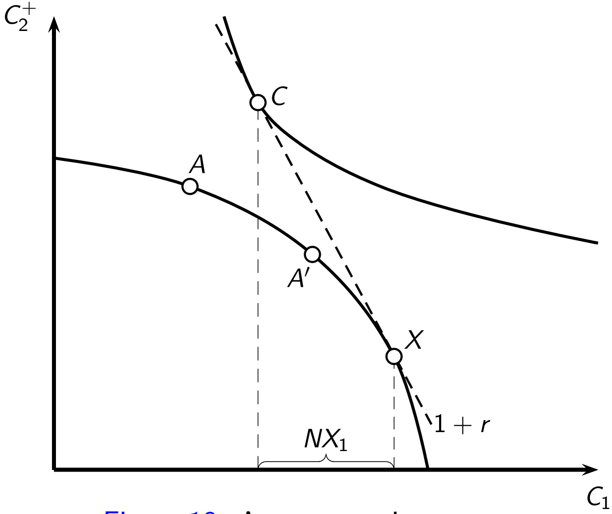



Figure 18: A net exporting country

# From autarky to open economy equilibrium

Production shifts in favor of current output by decreasing investment $\Delta I_1 < 0$. Additional output allows net exports.

Net exports today allow higher future consumption $\Delta C_2 > 0$ by future imports ($NX_2 = -(1 + r)NX_1$). Present consumption $C_1$ may shrink ($A'$) or increase ($A$) depending on the relative strength of income and substitution effect (plus output shift).

# Adding government consumption

# Adding government consumption

With government consumption, period utility has the following additive form: $u(C) + v(G)$. The budget constraint in the two period model is

$$C_1 + \frac{C_2}{1+r} = Y_1 - T_1 - I_1 + \frac{Y_2 - T_2 - I_2}{1+r},$$

where $T_t$ denotes taxes and $Y_t - T_t$ is 'disposable' income of the private sector in period $t$.

Goods market equilibrium in period $t$:

$$C_t + I_t + G_t + NX_t = Y_t \tag{26}$$

# Adding government consumption

Current account balance (recall (9),(10)):

$$
\begin{aligned}
CA_t &= NX_t + rB_t \\
&= Y_t + rB_t - C_t - G_t - I_t \\
&= \underbrace{Y_t + rB_t - C_t - T_t}_{S_t^P \text{ private savings}} + \underbrace{T_t - G_t}_{\text{public savings}} - I_t
\end{aligned}
$$

With a balanced budget $T_t = G_t$ of the public sector, private savings are equal to total savings ($S_t^P = S_t$) and

$$CA_t = \underbrace{S_t^P + T_t - G_t}_{S_t \text{ total savings}} - I_t \qquad (27)$$

$$B_{t+1} = B_t + S_t - I_t \qquad (28)$$

# Impact of $G$ in small open economy

Increase in $G_1(G_2)$ shifts transformation curve (PPF) for private sector leftward (downward).

In the following illustration (with a balanced budget of the government: $T_t = G_t$):

| | |
|---|---|
| Initial situation: | $G_1 = G_2 = 0$ and $NX_1 = NX_2 = 0$ |
| Shock 1: | $G_1 \uparrow$ |
| Shock 2: | $G_2 \uparrow$ |

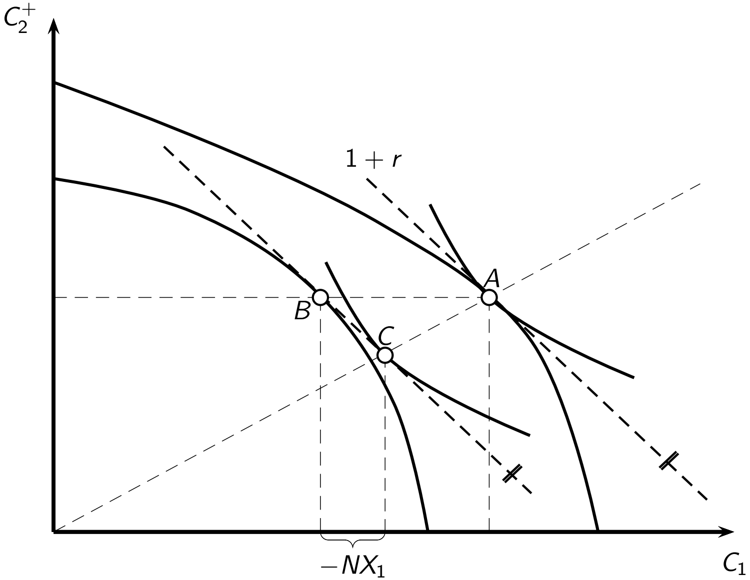



Figure 19: Impact of $G_1$ increase

# Impact of $G_1$ increase

- Private feasible output shifts from $A$ to $B$.
- Would decrease $C_1$ by the full amount of $G_1 = \overline{BA}$ leaving $C_2$ unaffected
- Individuals prefer $C$ by borrowing from abroad.

Impact of anticipated $G_2$ increase on next slide

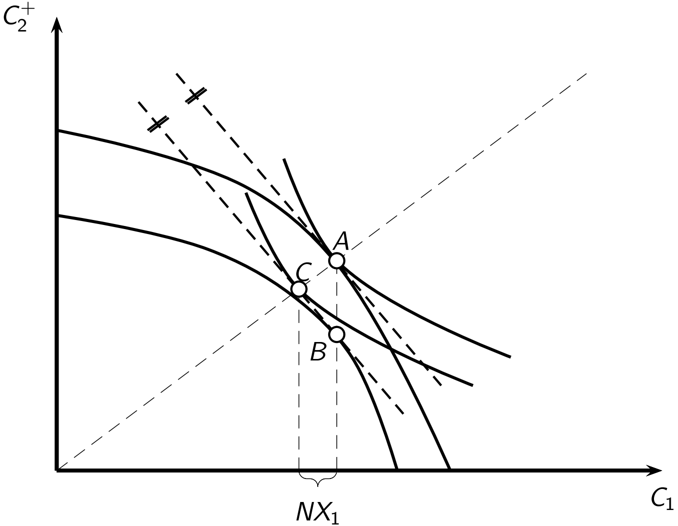



Figure 20: Impact of anticipated $G_2$ increase

Individuals "hedge" against tax $G_2$ by lending to *Foreign* in Period 1.

# Investment, savings and world interest rate in international equilibrium

# Investment, savings and world interest rate in international equilibrium

**The investment function**

Production function:

$$Y_t = A_t F(K_t)$$

$A_t$: Productivity parameter

Accumulation equation:

$$K_2 = K_1 + I_1$$

In the following, we consider a 2-period model with $B_1 = B_3 = 0$, $K_3 = 0$ and $G_t = T_t = 0$.

# The investment function

Condition for optimal capital input under perfect competition:

$$r = A_2 F'(K_1 + I_1) \quad (29)$$

(29) defines investment curve

$$I_1 = I(r/A_2),\ I^{'} < 0$$

The negative slope follows from $F'' < 0$.

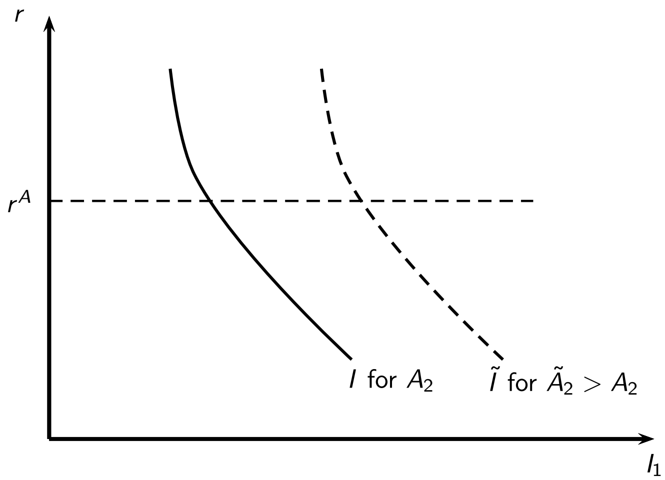



Figure 21: Investment curve and productivity shifts

Shifts in $A_1$ have no effect on investment since $K_1$ is already fixed from past decisions.

# The saving function

Reconsidering the endowment economy: From the endowment economy we know that $B_1 = B_3 = 0$ implies $S_1 = Y_1 - C_1 = NX_1(r)$.

Furthermore, we can note that, $dS_1/dr = dNX_1/dr = -dC_1/dr$.

To determine the impact of interest rate $r$ on savings (or, equivalently, $NX_1$), we can first look at the intertemporal Euler equation $u'(C_1) = (1+r)\beta u'(C_2)$.

Substituting the budget constraint $C_2 = (1+r)(Y_1 - C_1) + Y_2$ gives

$$u'(C_1) = (1+r)\beta u'\left((1+r)(Y_1 - C_1) + Y_2\right). \tag{30}$$

# The saving function

Implicitly differentiating (30) with respect to $r$ gives

$$\frac{dC_1}{dr} = \frac{\beta u'(C_2) + \beta(1+r)u''(C_2)(Y_1 - C_1)}{u''(C_1) + \beta(1+r)^2 u''(C_2)}. \quad (31)$$

Noting $u' > 0$, $u'' < 0$, it is immediate that $dNX_1(r)/dr = -dC_1/dr > 0$ if $C_1 > Y_1$ (or, equivalently, $NX_1 < 0$).

However, $dNX_1(r)/dr = -dC_1/dr < 0$ cannot be ruled out if $Y_1 > C_1$ (or, equivalently, $NX_1 > 0$) – see Figure 10.

# The saving function

Consumption in a model with capital accumulation and production

Substituting the budget constraint

$$C_2 = (1+r)[A_1 F(K_1) - C_1 - I_1] + A_2 F(K_1 + I_1) + K_1 + I_1$$

for $C_2$ in the Euler equation $u'(C_1) = (1+r)\beta u'(C_2)$, gives

$$u'(C_1) = (1+r)\beta u' \{(1+r)[A_1 F(K_1) - C_1 - I_1] + A_2 F(K_1 + I_1) + K_1 + I_1\}. \quad (32)$$

Implicitly differentiating with respect to $r$ yields

$$\frac{dC_1}{dr} = \frac{\beta u'(C_2) + \beta(1+r)u''(C_2)[A_1 F(K_1) - C_1 - I_1]}{u''(C_1) + \beta(1+r)^2 u''(C_2)} + \frac{\beta(1+r)u''(C_2)\{A_2 F'(K_1 + I_1) - r\}\partial I/\partial r}{u''(C_1) + \beta(1+r)^2 u''(C_2)}.$$

# The saving function

Accounting for $A_2 F'(K_1 + I_1) = r$ further implies

$$\frac{dC_1}{dr} = \frac{\beta u'(C_2) + \beta(1+r)u''(C_2)\left[A_1 F(K_1) - C_1 - I_1\right]}{u''(C_1) + \beta(1+r)^2 u''(C_2)}. \qquad (33)$$

Hence, the derivative in (33) is precisely the same as the derivative in (31), but with $Y_1 - C_1$ replaced by the date 1 current account for an investment economy with $B_1 = 0$: $A_1 F(K_1) - C_1 - I_1$.

That means that, given current account balances, the slope of the saving schedule is the same as for the endowment economy!

# The saving function

An intuition for this result

The symmetry in the reaction of savings to interest rate adjustments in the endowment and the investment economy is a consequence of the *envelope theorem*.

The first-order condition for profit-maximizing investment ensures that a small deviation from optimum investment does not alter the present value of national output, evaluated at the world interest rate.

Consequently, at the margin, the investment adjustment $\partial I_1/\partial r$ has no effect on net lifetime resources, and hence no effect on consumption response.

# The saving function

From consumption to saving

As noted above, savings in period 1 are given by $S_1 = Y_1 - C_1$ or, equivalently, $S_1 = A_1 F(K_1) - C_1$. Hence, we can write savings as function of $r$, $A_1$, $A_2$ and $\beta$:

$$S_1 = S(r, A_1, A_2, \beta),$$

with $\partial S_1 / \partial r > 0$ in the regular (*non-perverse*) case.

Saving curve and productivity shift

An increase of $A_t$ has analogous effects to an increase of $Y_t$ in endowment economy.

- According to slide 52 a rise in $Y_1$ shifts the $S$-curve to the right. A rise in $Y_2$ shifts the $S$-curve to the left.
- Rising impatience (a fall in $\beta$) also shifts the saving curve to the left.

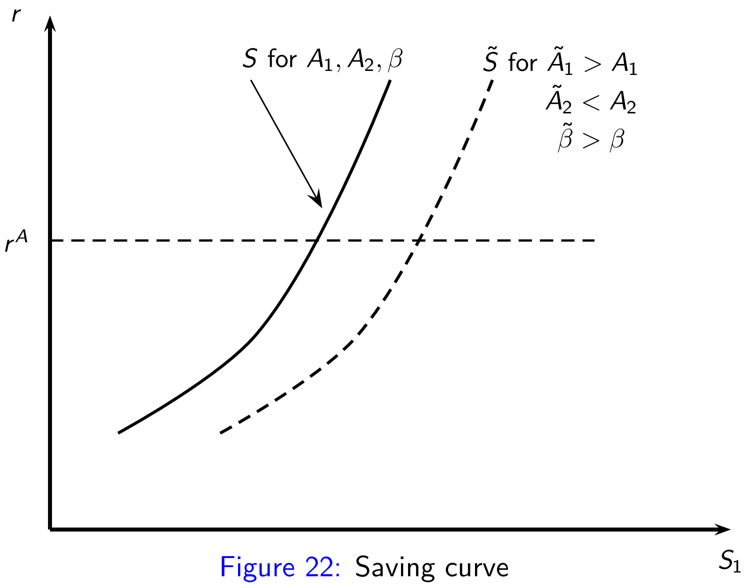



Figure 22: Saving curve

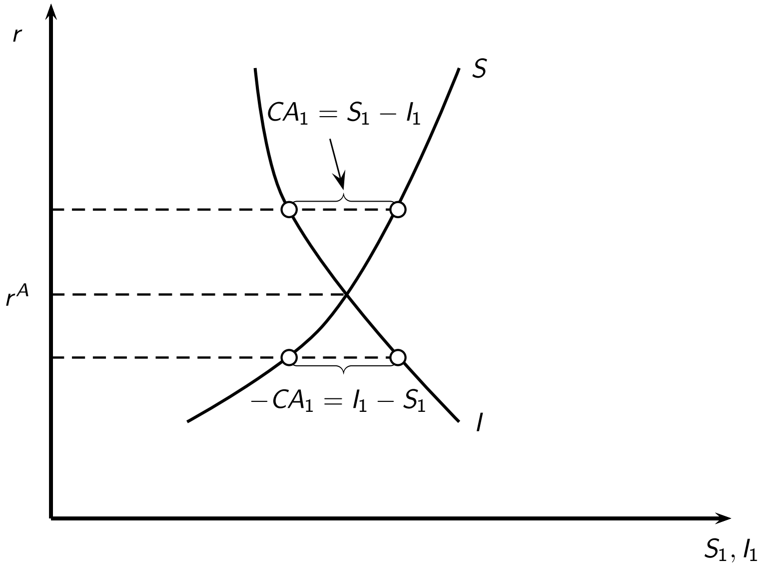



Figure 23: Investment, savings, and current account

# International equilibrium in a two-region world

**The Metzler diagrams**

World equilibrium requires

$$CA_1 + CA_1^* = 0$$

i.e.

$$S_1 - I_1 = -\left(S_1^* - I_1^*\right)$$

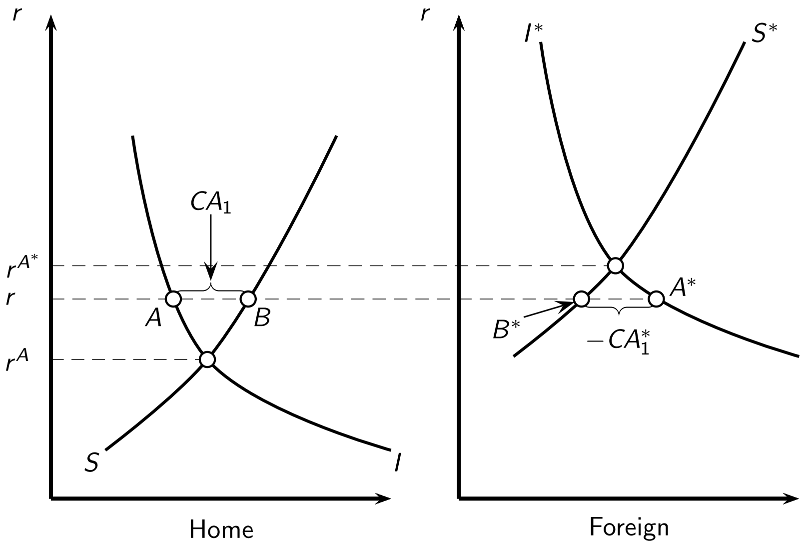



Figure 24: World equilibrium interest rate $r^A < r < r^{A*}$

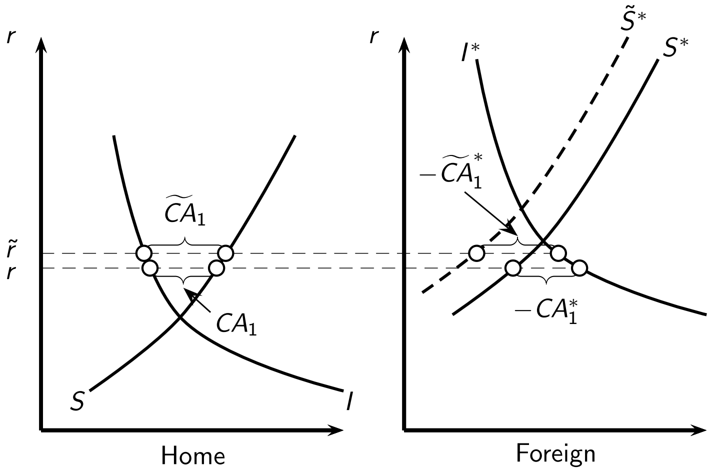



Figure 25: Impact of rising impatience in *Foreign* ($\beta^* \downarrow$)

World interest rate rises and current account $CA_1$ from *Home* to *Foreign* increases. Investment decreases in both regions.

# Impact of positive productivity shock in *Foreign*

Consider a productivity shock of the form $A_2^* \uparrow$

World interest rate rises

| | |
|---|---|
| In *Home* | Investment falls<br>Saving and $CA_1$-surplus rise |
| In *Foreign* | Investment reaction ambiguous<br>$CA_1^*$-deficit rises |

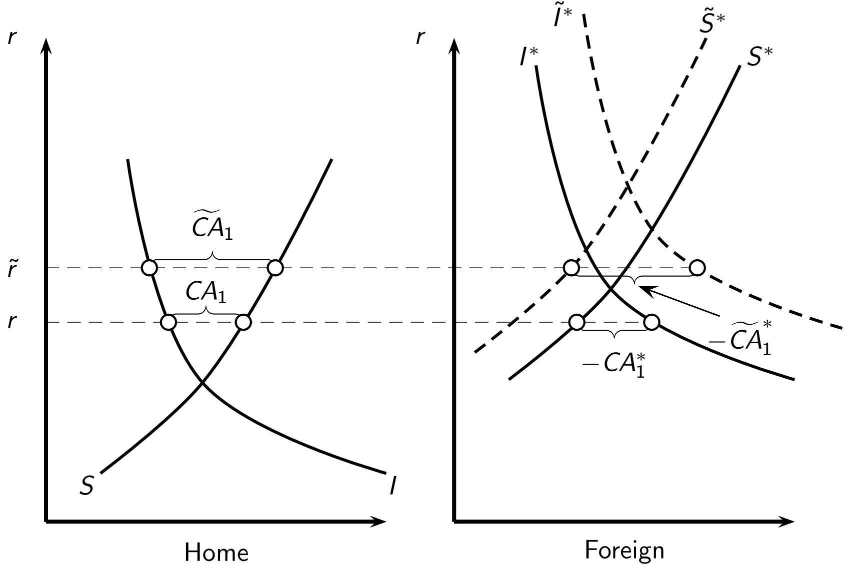



Figure 26: Impact of positive productivity shock in Foreign

# Stability and Marshall-Lerner condition

The world equilibrium condition $CA_t + CA_t^* = 0$ is

$$S_t(r) + S_t^*(r) = I_t(r) + I_t^*(r) \tag{34}$$

(Use $CA_t = S_t - I_t$)

As addressed in Figure 10, the saving curve may be backward bending, so that multiple equilibria and unstable equilibria cannot be excluded.

# Stability and Marshall-Lerner condition

(Walrasian) stability condition: A market is stable in the Walrasian sense if a small increase in the price of the good traded there causes excess supply, while a small decrease causes excess demand.

The stability condition defining Walrasian stability in the market for world savings is that a small rise in $r$ should lead to an excess supply of savings:

$$\frac{d\left[S_1(r)+S_1^*(r)\right]}{dr} > \frac{d\left[I_1(r)+I_1^*(r)\right]}{dr} \tag{35}$$

Stability guarantees that market forces tend to eliminate imbalances resulting from small disturbances of international equilibrium.

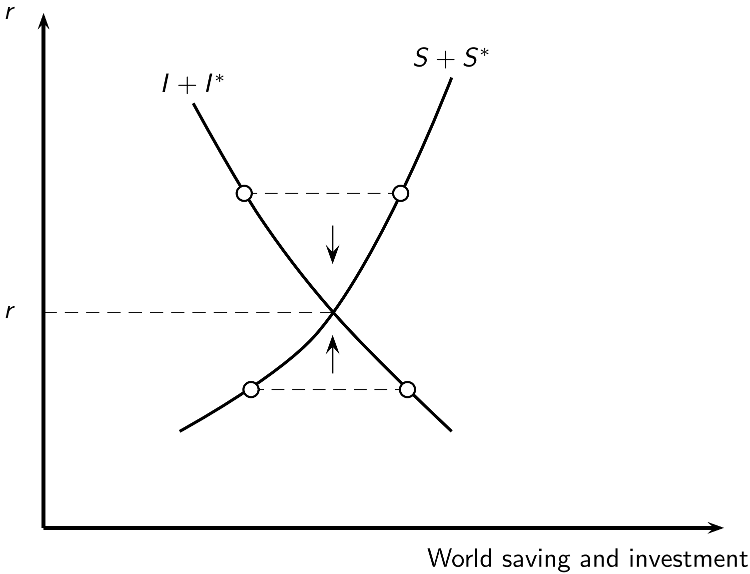



Figure 27: Savings and Investment

# Stability and Marshall-Lerner condition

For $B_1 = B_3 = 0$ national accounting identities imply

$$\begin{aligned} NX_1 &= CA_1 = S_1 - I_1 \\ NX_1^* &= CA_1^* = S_1^* - I_1^* \end{aligned}$$

Moreover (see (11)),

$$NX_1^* + \frac{NX_2^*}{1+r} = 0$$

# Stability and Marshall-Lerner condition

Using this in international equilibrium condition (34), we get

$$S_1 - I_1 + S_1^* - I_1^* = NX_1 - \frac{NX_2^*}{1+r}$$

Thus (35) is equivalent to

$$\frac{d\left[NX_1(r) - \frac{NX_2^*(r)}{1+r}\right]}{dr} > 0 \tag{36}$$

# Stability and Marshall-Lerner condition

$$
\begin{aligned}
\frac{d\left[NX_1(r) - \dfrac{NX_2^*(r)}{1+r}\right]}{dr} &= NX_1' - \frac{NX_2^{*'}(1+r) - NX_2^*}{(1+r)^2} \\
&= \frac{NX_2^*}{(1+r)^2}\left[\frac{(1+r)NX_1'}{NX_1}\frac{NX_1(1+r)}{NX_2^*}\right. \\
&\quad \left. - \frac{NX_2^{*'}(1+r)}{NX_2^*} + 1\right]
\end{aligned}
$$

# Stability and Marshall-Lerner condition

In equilibrium $NX_1(1+r) = -NX_2 = NX_2^*$. Thus the square bracket is negative (positive) if

$$\underbrace{-\frac{(1+r)NX_1'}{NX_1}}_{\eta} + \underbrace{\frac{(1+r)NX_2^{*\prime}}{NX_2^*}}_{\eta^*} > (<)\, 1 \tag{37}$$

If Home is net importer today ($NX_1 < 0$), then $NX_2^* < 0$ and stability condition (35) is equivalent to

$$\eta + \eta^* > 1. \tag{38}$$

# Stability and Marshall-Lerner condition

Interpretation ($NX_1 < 0$)

$\eta$ is the (absolute value of) negative import elasticity of Home with respect to price $1 + r$ of current consumption. $\eta^*$ is the (positive) elasticity of Foreign's future imports. (38) is the intertemporal analogue to the so-called *Marshall-Lerner* condition.

Remark

When *Home* happens to be the exporter in period 1, rather than the importer, (38) still characterizes the Walras-stable case, but with import elasticities defined so that Home's and Foreign's role are interchanged.