Advanced (International) Macroeconomics

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Intertemporal Trade and Current Account

2-Period Model

Within periods:
Inter- and intraindustry trade (see trade theory from Ricardo, Heckscher/Ohlin to new trade theories with imperfect markets). Gains from international labor division (comparative advantages), exploitation of economies of scale or intensified competition.

Across periods:
Intertemporal trade. Gains from international borrowing and lending.

Intertemporal choice - preferences

$$U = u(c_1) + \beta u(c_2), \quad 0 < \beta < 1 \tag{1}$$

- u period (instantaneous) utility
- $u' > 0, u'' < 0, \lim_{c \to 0} u'(c) = \infty$
- β subjective discount factor (time-preference parameter)

Marginal rate of substitution

$$MRS \left(\equiv -\frac{dc_2}{dc_1}\bigg|_{U=const} \right) = \frac{u'(c_1)}{\beta u'(c_2)} \tag{2}$$

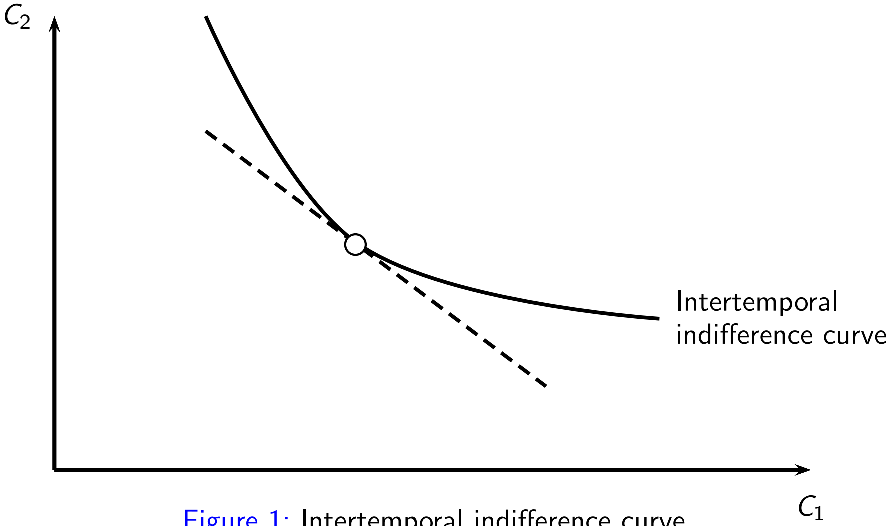


Figure 1: Intertemporal indifference curve

Example utility functions

- Example 1: $u(c_t) = \ln c_t$
- Example 2: $u(c_t) = \dfrac{c_t^{1-1/\sigma}}{1-1/\sigma} \quad , \qquad 0 < \sigma (\neq 1).$

(Isoelastic utility functions)

Intertemporal budget constraint

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} \tag{3}$$

- r interest rate (if there are variable interest rates, r_{t+1} denotes interest rate from t to $t+1$. Then $\frac{c_2}{1+r_2}$ etc.).
- y_t endowment (income) in period t.

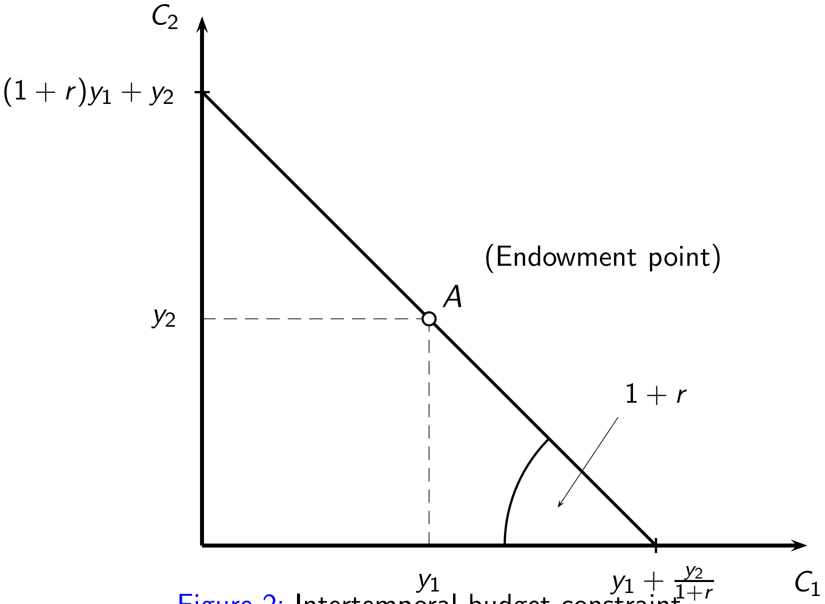


Figure 2: Intertemporal budget constraint

Optimal intertemporal choice

$$\max \quad U \qquad \text{s.t.} \quad (3)$$

Lagrange-Function:

$$\mathcal{L} = u(c_1) + \beta u(c_2) + \lambda \left(y_1 + \frac{y_2}{1+r} - c_1 - \frac{c_2}{1+r} \right)$$

First-order conditions

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_1} &= 0 \implies u'(c_1) = \lambda \\
\frac{\partial \mathcal{L}}{\partial c_2} &= 0 \implies \beta u'(c_2) = \frac{\lambda}{1+r} \\
\frac{\partial \mathcal{L}}{\partial \lambda} &= 0 \implies (3)
\end{aligned}$$

combine to the so-called *intertemporal Euler equation*:

$$u'(c_1) = (1+r)\beta u'(c_2) \tag{4}$$

Eq. (4) determines how consumption needs to be allocated intertemporally in order to maximize utility at a given interest rate r.

Interpretation of Euler Equation

The intertemporal choice is optimal if there are no gains from intertemporal reallocation. Equation (4) is equivalent to

$$MRS = 1 + r \tag{5}$$

where MRS is given by (2)

Equilibrium in an endowment economy

Endowment economies have no capital accumulation and no production.

Aggregate supply (with symmetric agents):
$Y_t = y_t N_t,\ t = 1, 2$ where N_t is the population size in period t. In the following, N_t is normalized to 1 so that $Y_t = y_t$.

Aggregate demand:
$C_t = c_t,\ t = 1, 2$

In **closed economy** (autarky):

$$C_t = Y_t, \ t = 1, 2 \tag{6}$$

In **open economy**:

$$C_t = Y_t + r_t B_t - CA_t \tag{7}$$

where B_t is the value of *net foreign assets* inherited from period $t-1$ and CA_t is the *current account balance* (Ertragsbilanz, auch Leistungsbilanz).

By definition,

$$CA_t = B_{t+1} - B_t \tag{8}$$

Remarks on national accounting

- *Gross domestic product* (GDP) (Bruttoinlandsprodukt): Y_t
- *Gross national product* (GNP) (Bruttosozialprodukt): $Y_t + r_t B_t$ i.e.
    - GNP=GDP+net international factor payments
    - net international factor payments here only includes interest and dividend earnings on net foreign assets
    - but no workers remittances
- *Trade balance* (goods and services): Net exports NX_t

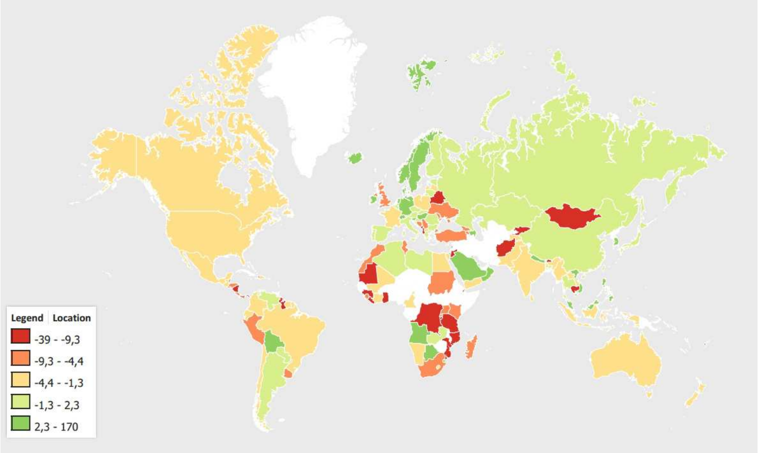
Remarks on national accounting

- *Capital account balance* (Kapitalverkehrsbilanz, auch Kapitalbilanz; includes financial account balance): Net sales of foreign assets: $-(B_{t+1} - B_t)$
- *Balance of payments* (Zahlungsbilanz): $NX_t + r_t B_t = B_{t+1} - B_t$
- *Current account balance* (Ertragsbilanz, auch Leistungsbilanz):

$$\text{within period perspective: } CA_t = NX_t + r_t B_t \tag{9}$$

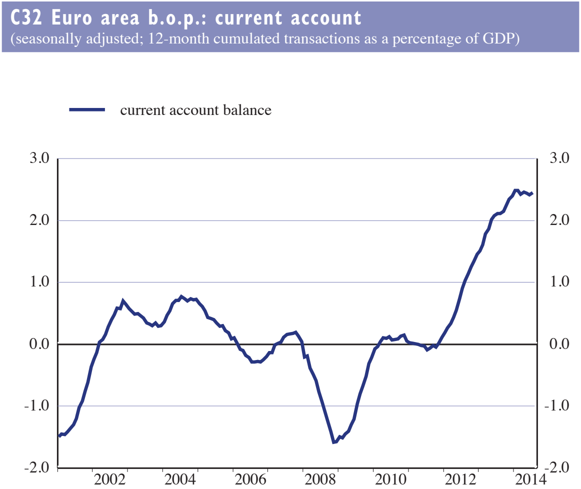
$$\text{intertemporal perspective: } CA_t = B_{t+1} - B_t \tag{10}$$

Remarks on national accounting



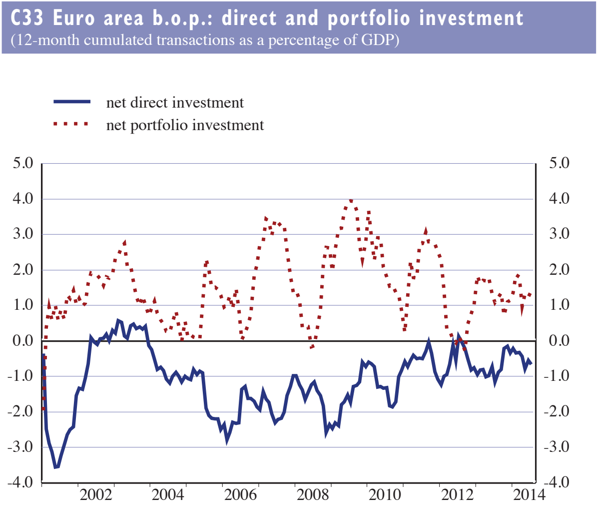
2013 current account balance in % of GDP
Source: Worldbank database

Remarks on national accounting



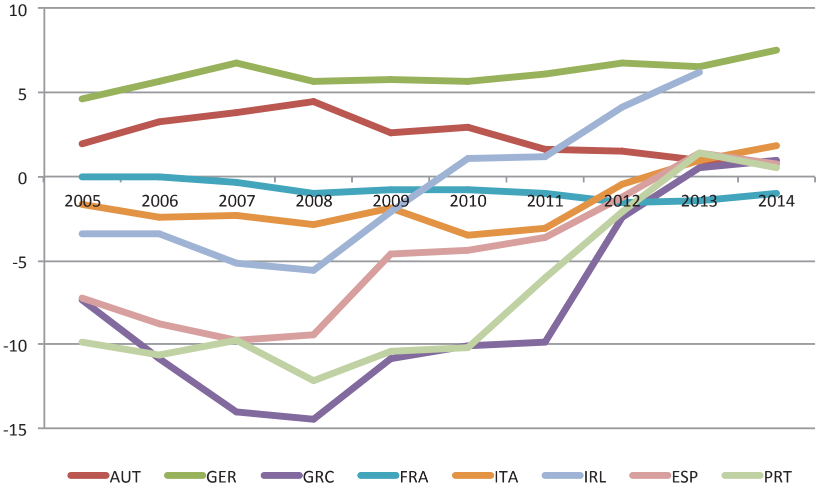
Source: Euro Area statistics online

Remarks on national accounting



Source: Euro Area Statistics online

Remarks on national accounting



Current account figures for selected Euro member states
Source: World Development Indicators

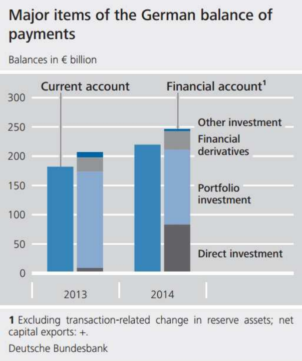
Remarks on national accounting

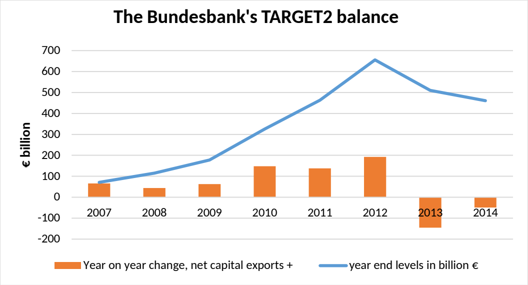
€ billion			
Item	2012r	2013r	2014r
I Current account (balance)	+ 187.3	+ 182.0	+ 219.7
1 Goods[1]	+ 196.6	+ 207.6	+ 229.3
Exports (fob)	1,074.1	1,083.5	1,123.8
Imports (fob)	877.5	875.9	894.5
Memo item			
Foreign trade[2]	+ 189.8	+ 195.0	+ 216.9
Exports (fob)	1,095.8	1,093.1	1,133.5
Imports (cif)	905.9	898.2	916.6
2 Services (balance)[3]	− 35.9	− 44.8	− 39.1
of which			
Travel (balance)	− 35.4	− 37.7	− 36.8
3 Primary income (balance)	+ 66.8	+ 60.2	+ 66.9
of which			
Investment income (balance)	+ 62.2	+ 58.0	+ 65.0
4 Secondary income (balance)	− 40.1	− 41.1	− 37.4
II Balance on capital account	+ 1.4	+ 1.1	+ 2.8
III Balance on financial account[4]	+ 157.5	+ 207.9	+ 243.8
1 Direct investment	+ 35.6	+ 9.0	+ 83.2
2 Portfolio investment	+ 54.8	+ 164.5	+ 127.7
3 Financial derivatives[5]	+ 24.4	+ 24.3	+ 31.8
4 Other investment[6]	+ 41.4	+ 9.3	+ 3.7
5 Reserve assets[7]	+ 1.3	+ 0.8	− 2.6
IV Errors and omissions[8]	− 31.3	+ 24.8	+ 21.3

1 Excluding freight and insurance costs of foreign trade. **2** Special trade according to the official foreign trade statistics (source: Federal Statistical Office). **3** Including freight and insurance costs of foreign trade. **4** Increase in net external position: + / decrease in net external position: -. **5** Balance of transactions arising from options and financial futures contracts as well as employee stock options. **6** Includes in particular loans and trade credits as well as currency and deposits. **7** Excluding allocation of special drawing rights and excluding changes due to value adjustments. **8** Statistical errors and omissions, resulting from the difference between the balance on the financial account and the balances on the current and the capital account.
Deutsche Bundesbank

Balance of payments of Germany
Source: Deutsche Bundesbank: Monthly Report, March 2015

Remarks on national accounting





Source: Deutsche Bundesbank: Monthly Reports, March 2015

Remarks on national accounting

Long-run impact of short-run imbalances:

$$CA_t = NX_t + r_t B_t \text{ and } CA_t = B_{t+1} - B_t$$

imply

$$B_{t+1} = NX_t + (1 + r_t)B_t$$

Repeating the argument for B_{t+2} we get

$$B_{t+2} = NX_{t+1} + (1 + r_{t+1})NX_t + (1 + r_t)(1 + r_{t+1})B_t$$

Remarks on national accounting

In a $T+1$-period world with $r_t = r$ (inheriting B_t and leaving B_{t+T+1}):

$$\begin{aligned} B_{t+T+1} &= (1+r)^{T+1}B_t + (1+r)^T NX_t + \dots \\ &\quad +(1+r)NX_{t+T-1} + NX_{t+T} \end{aligned}$$

$$\iff$$

$$\left(\frac{1}{1+r}\right)^T B_{t+T+1} = (1+r)B_t + \sum_{s=t}^{t+T}\left(\frac{1}{1+r}\right)^{s-t} NX_s \qquad (11)$$

Terminal condition

$$B_{t+T+1} = 0$$

implies

$$\sum_{s=t}^{t+T} \left(\frac{1}{1+r} \right)^{s-t} NX_s = -(1+r)B_t$$

For instance, for $B_3 = B_1 = 0$ (temporary imbalance[1]):

$$\begin{aligned} NX_1 + \frac{NX_2}{1+r} &= -(1+r)B_1 = 0 \\ CA_1 + CA_2 &= 0 \end{aligned}$$

and

$$\begin{aligned} CA_1 &= B_2 - B_1 = B_2 \\ CA_2 &= B_3 - B_2 = -B_2 \\ NX_1 &= CA_1 - rB_1 = CA_1 = B_2 \\ NX_2 &= CA_2 - rB_2 = -(1+r)B_2 \end{aligned}$$

[1] if interested in more long-run dynamics, see Obstfeld/Rogoff Chapter 2

Equilibrium in a closed economy

Goods market equilibrium

$$C_t = Y_t,\ t = 1, 2$$

and optimal consumption choice (cf. intertemporal Euler equation)

$$u'(C_1) = (1 + r)\beta u'(C_2)$$

give us *the autarky real interest rate*

$$1 + r^A = \frac{u'(Y_1)}{\beta u'(Y_2)} \tag{12}$$

(Budget constraint (3) is obviously fulfilled for $C_t = Y_t$.)

- $1 + r^A$ is the willingness to pay for present consumption.
- Virtual price in closed economy without investment possibilities.
- Relevant when opening up.

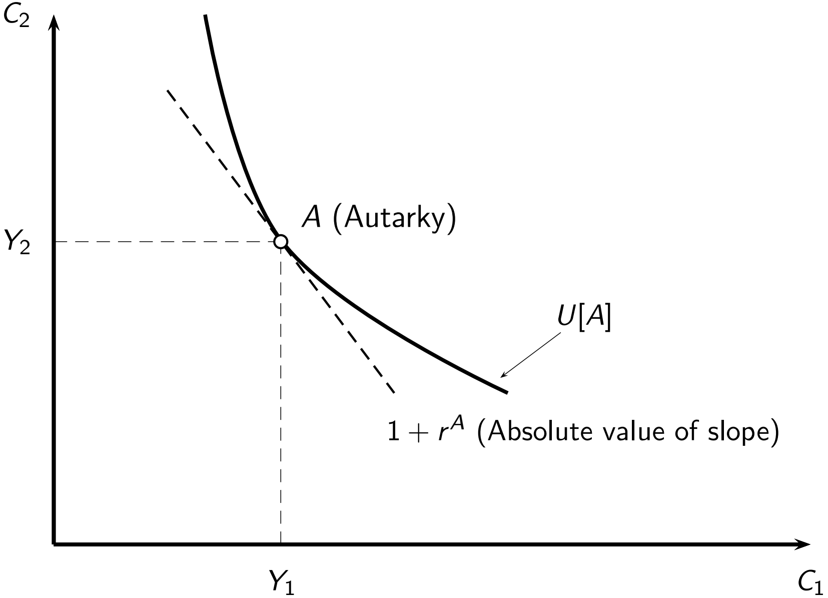


Figure 3: Autarky equilibrium

Effect of time preference on r^A

$\beta > \widetilde{\beta}$ implies

$$MRS(Y_1, Y_2) \quad < \quad \widetilde{MRS}(Y_1, Y_2)$$

$$\text{where } MRS(Y_1, Y_2) \quad \equiv \quad \frac{u'(Y_1)}{\beta u'(Y_2)}$$

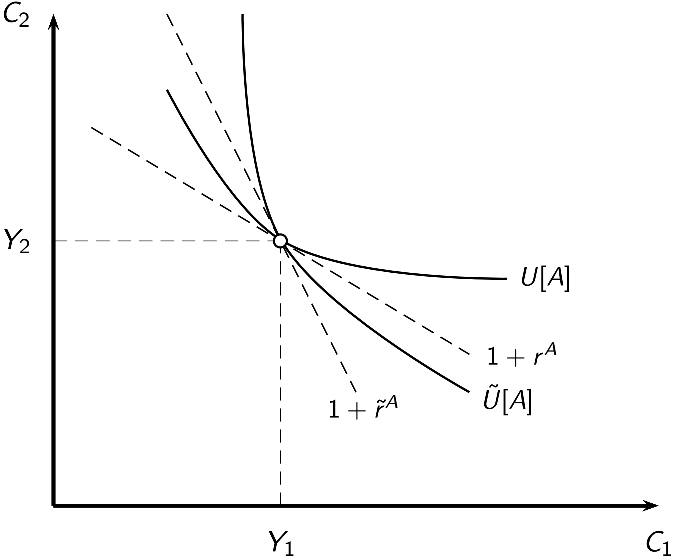


Figure 4: r^A rises with impatience.

Effect of output changes on r^A

Assume linear consumption expansion path
$(MRS(\lambda Y_1, \lambda Y_2) = MRS(Y_1, Y_2))$

- r^A rises if positive output shock is expected.
- No change of r^A if present and future output rise pari passu.

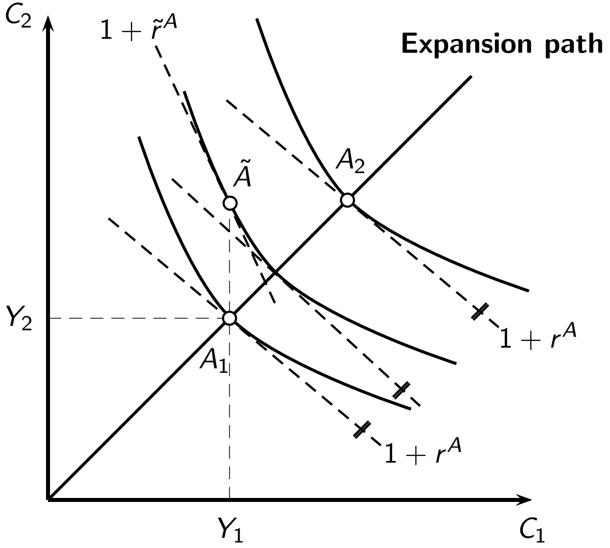


Figure 5: Effect of output changes on r^A

Equilibrium in a small open economy

- 2 periods: $B_1 = B_3 = 0$ i.e. $NX_1 + \frac{NX_2}{1+r} = 0$
- r exogenously given by the world market

Intertemporal equilibrium allocation C_1, C_2 determined by:

$$u'(C_1) = (1+r)\beta u'(C_2) \quad (13)$$

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r} \quad (14)$$

Special case

If subjective discount factor is equal to market discount factor

$$\beta = \frac{1}{1+r},$$

the solution of (13) & (14) is given by

$$C_1 = C_2 \equiv \overline{C} \quad (15)$$

$$\overline{C} = \frac{(1+r)Y_1 + Y_2}{2+r} \quad (16)$$

For $\beta < \dfrac{1}{1+r}$ the allocation is biased in favor of C_1.

Open vs. closed economy equilibrium

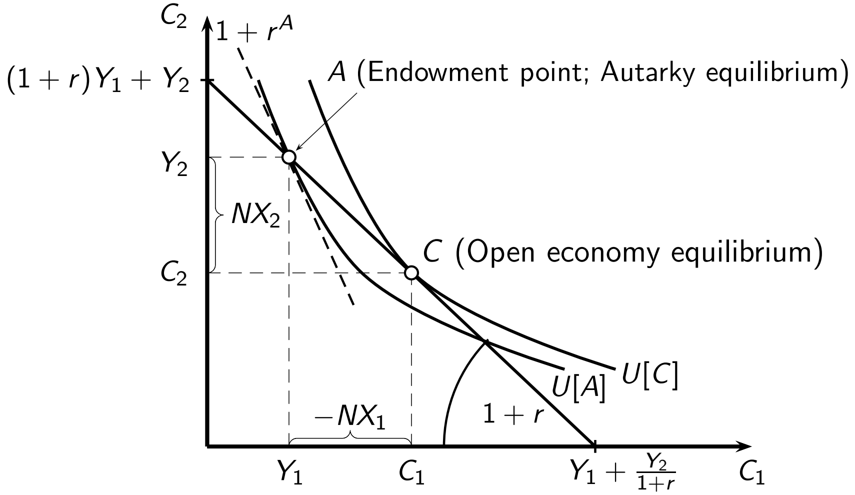


Figure 6: Comparing open and closed equilibrium if $r_A > r$

- Access to international capital markets allows intertemporal income shifting.
- Here from future to present (borrowing) since $r < r^A$.
- Gains from intertemporal trade $U[C] > U[A]$
- Debt from current net imports $NX_1 = Y_1 - C_1 < 0$ must be paid back by future net exports $NX_2 = -(1+r)NX_1 = Y_2 - C_2 > 0$

Implications for trade flows and capital account

According to (7), (9) and (10):

$$\begin{aligned} C_1 = \quad Y_1 + rB_1 - CA_1 &= Y_1 + (1+r)B_1 - B_2 \\ &= Y_1 - NX_1 \end{aligned}$$

$$\begin{aligned} C_2 = \quad Y_2 + rB_2 - CA_2 &= Y_2 + (1+r)B_2 - B_3 \\ &= Y_2 - NX_2 \end{aligned}$$

In 2-period world: $B_1 = B_3 = 0$

Trade flows

$$\begin{aligned} C_1 - Y_1 &= -NX_1 \\ Y_2 - C_2 &= (1+r)(C_1 - Y_1) = NX_2 \end{aligned}$$

Net foreign assets

$$\begin{aligned} B_2 &= -(C_1 - Y_1) \\ Y_2 - C_2 &= -(1+r)B_2 = (1+r)(C_1 - Y_1) \end{aligned}$$

"Long-run" effects of short-run trade deficit

If endowment expectations are wrong, the associated short run trade deficit may have long-run implications (3 or more periods, $B_1 = 0$).

- $CA_1 = NX_1 = B_2$
- $CA_2 = rB_2 = rNX_1$
- $B_3 = CA_2 + B_2 = (1 + r)NX_1$

$B_{t+1} = B_t, \; t \geqslant 3$ requires $CA_t = NX_t + rB_t = 0$

$$\Longrightarrow \text{For } t \geqslant 3\text{: } NX_t = -r(1 + r)NX_1 \Longrightarrow C_{t(\geqslant 3)} < Y_t$$

$B_4 = 0$ requires $CA_3 = -B_3$

$$\Longrightarrow NX_3 = -(1 + r)^2 NX_1 \Longrightarrow \widetilde{C}_3 < C_{t(\geqslant 3)} < Y_3$$

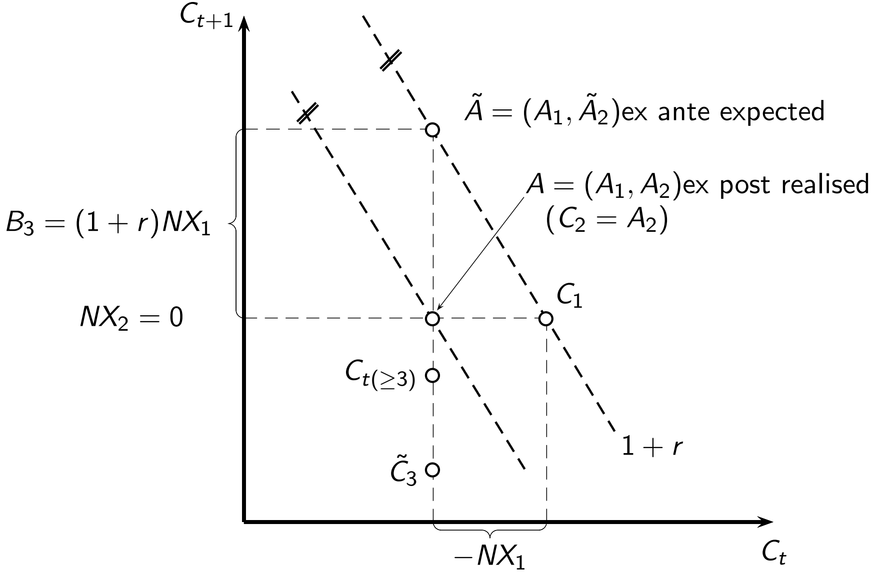


Figure 7: "Long-run" effects of trade deficit (3 per., $B_1 = 0$)

Intertemporal trade pattern, savings and international equilibrium

Case $r < r^A$

(see figure 6): Implies $NX_1 < 0,\ NX_2 > 0$, given $B_1 = B_3 = 0$.

- The country is a net importer in period 1, net exporter in period 2.
- $1 + r$ is the price of present consumption (here the import good) in terms of future consumption (export good).

Terms of trade

$$\text{Terms of trade} = \frac{\text{price of exports}}{\text{price of imports}} = \frac{1}{1+r}$$

decline in r $\Rightarrow$ i) terms of trade improve

$\Rightarrow$ positive income and wealth effect on C_1

ii) substitution effects also favors C_1

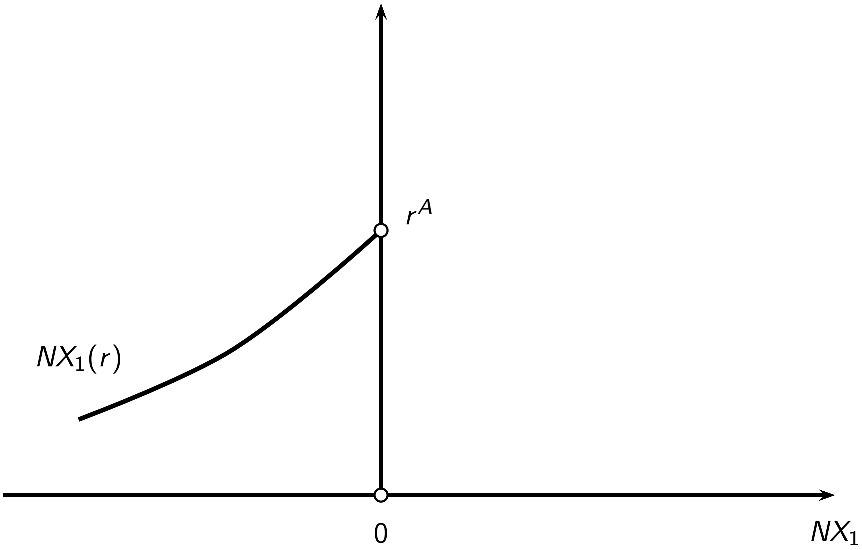


Figure 8: Net exports and interest rate if $r < r^A$

Case $r > r^A$

(see figure 9): Implies $NX_1 > 0,\ NX_2 < 0$, given $B_1 = B_3 = 0$.

- Country is net exporter of present output, terms of trade $1 + r$

$$\begin{aligned} \textit{rise in } r \quad &\Rightarrow \quad \text{i) positive terms of trade effect on } C_1 \\ &\Rightarrow \quad \text{ii) negative substitution effect on } C_1 \end{aligned}$$

in sum, the NX_1-reaction is ambiguous.

Case $r > r^A$

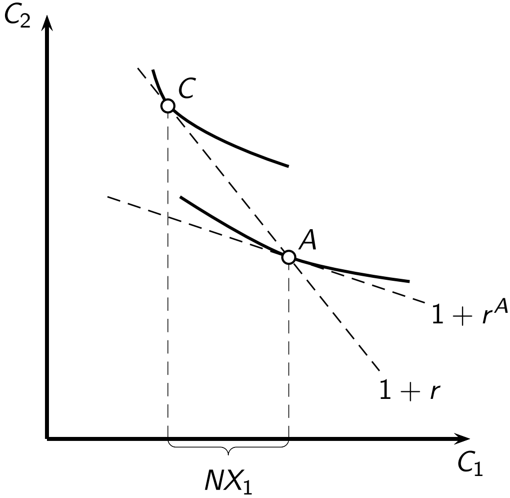


Figure 9: Intertemporal trade with $r > r^A$

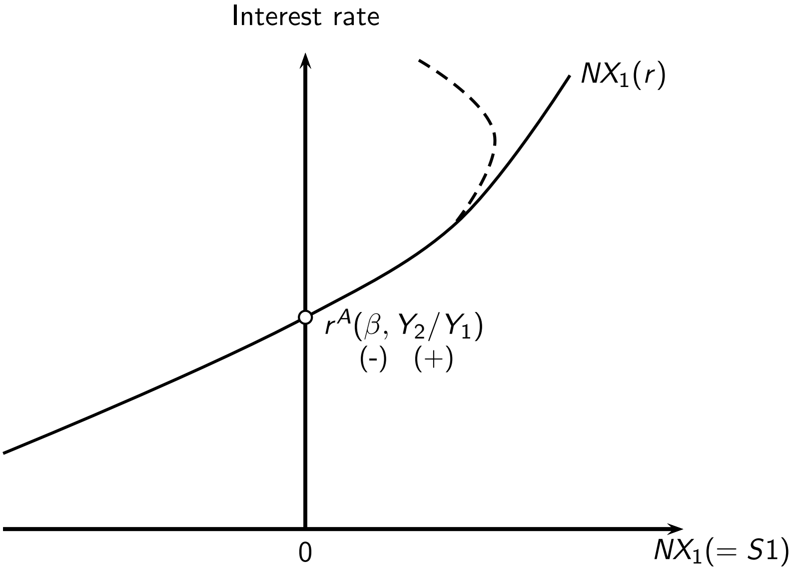


Figure 10: Net exports and interest rate if $r > r^A$

The position of the curve is fixed by r^A.

Remember: r^A declines with β and (for linear expansion path of consumption) rises with Y_2/Y_1.

Moreover: For $B_1 = 0$, $CA_1 = NX_1$ and thus $S_1 \equiv Y_1 + rB_1 - C_1 = NX_1$

International equilibrium

Two country world:

$$\begin{aligned} \text{Home:} \quad & r^A(\beta, Y_2/Y_1) \\ \text{Foreign:} \quad & r^{A^*}(\beta^*, Y_2^*/Y_1^*) \end{aligned}$$

integrated world is like closed economy with goods market equilibrium condition

$$C_t + C_t^* = Y_t + Y_t^*$$

Using $C_t + NX_t = Y_t,\ C_t^* + NX_t^* = Y_t^*$, we get

$$NX_t + NX_t^* = 0 \tag{17}$$

This determines world interest rate r.

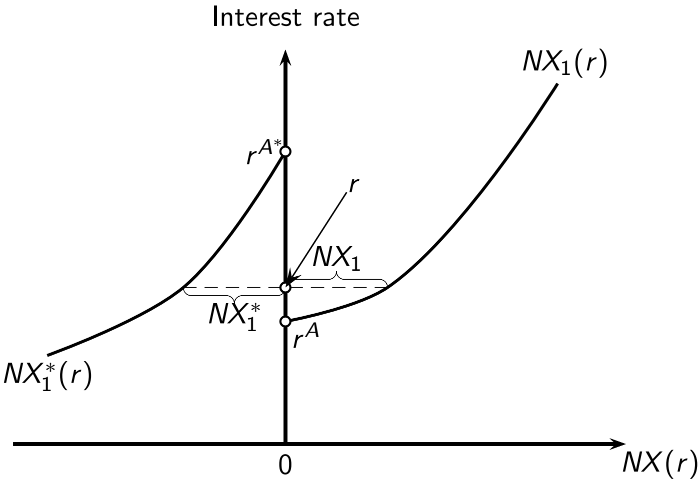


Figure 11: Equilibrium world interest rate if $r^{A^*} > r^A$

If increasing impatience (β or $\beta^* \downarrow$) or rising future output (Y_2/Y_1 or $Y_2^*/Y_1^* \uparrow$) raise r^A or r^{A^*} the equilibrium world interest rate rises, ceteris paribus. This

- worsens terms of trade for net importer Foreign
- improves terms of trade for net exporter Home

Capital accumulation and production

Basic model assumptions

Production function:

$$Y_t = F(K_t)$$

Properties:

- $F(0) = 0$
- $F' > 0$
- $F'' < 0$

Inada conditions:

- $\lim_{K \to 0} F'(K) = \infty$
- $\lim_{K \to \infty} F'(K) = 0$

Since $N_t = 1$, level of capital stock K_t and capital intensity $k_t = K_t / N_t$ coincide.

Capital accumulation:

$$K_{t+1} = K_t + I_t \tag{18}$$

(Depreciation ignored, K can be eaten up, i.e. $I_t = -K_t$.)

Capital demand under perfect competition:

$$r_t = F'(K_t) \tag{19}$$

Wages (labor demand) under perfect competition:

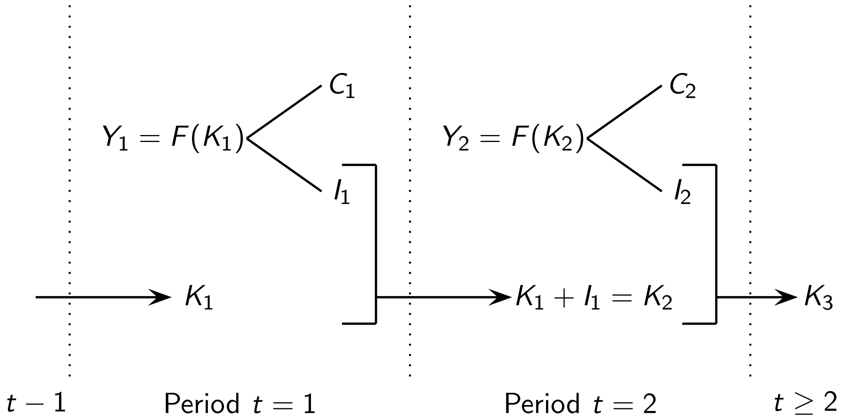
$$W_t = F(K_t) - r_t K_t \tag{20}$$

Closed economy with capital accumulation and production

Intertemporal production and investment

Goods market equilibrium

$$C_t + I_t = Y_t \tag{21}$$



Intertemporal transformation curve ("Production possibilities Frontier" PPF)

$$\begin{aligned} C_2 + K_3 &= F(K_2) + K_2 \\ &= F\left[K_1 + \underbrace{F(K_1) - C_1}_{I_1}\right] + K_1 + F(K_1) - C_1 \end{aligned} \tag{22}$$

Intertemporal PPF:

$$C_2^+ \equiv C_2 + K_3$$

$$C_2^+ = F\left(\underbrace{K_1 + F(K_1) - C_1}_{K_2}\right) + \underbrace{K_1 + F(K_1) - C_1}_{K_2}$$

$$\frac{dC_2^+}{dC_1} = -\left[1 + F'\left(\underbrace{K_1 + F(K_1) - C_1}_{K_2}\right)\right] < 0$$

$$\frac{d^2 C_2^+}{dC_1^2} = F''(K_2) < 0$$

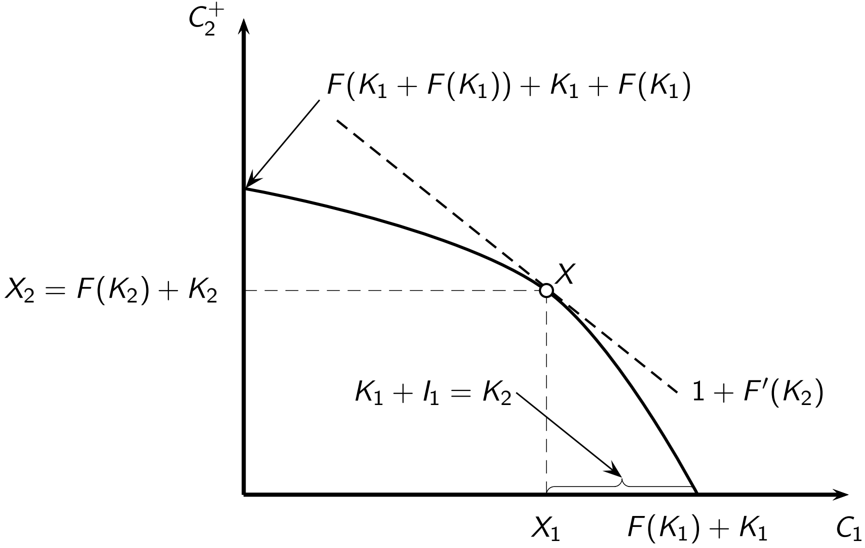
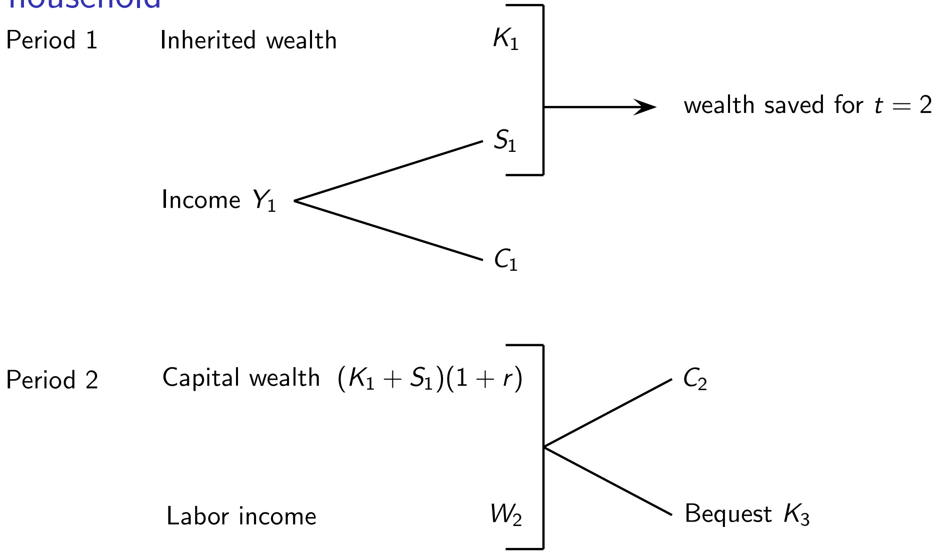


Figure 12: Intertemporal production possibilities frontier

Intertemporal budget constraint of representative household



Intertemporal consumption possibility line (CPL)

Intertemporal budget constraint:

$$C_2 + K_3 = W_2 + (K_1 + Y_1 - C_1)(1 + r) \tag{23}$$

Since $W_2 = F(K_2) - rK_2$ and $K_1 + Y_1 - C_1 = K_2$ (23) is consistent with (22).

That means: Savings behavior of households leads to a point on the economy's PPF.

The question is: which point?

Optimal intertemporal choice

K_3- choice depends on the "bequest" motive. Can be captured by

$$u(C_1) + \beta u(C_2^+)$$

$$\begin{aligned} C_2^+ &= C_2 + K_3 \ldots \text{ "bequest" motive} \\ C_2^+ &= C_2 \ldots \text{ no "bequest" motive} \end{aligned}$$

$$\max\ u(C_1) + \beta u(C_2^+) \text{ s.t. } C_2^+ = W_2 + (K_1 + Y_1 - C_1)(1 + r)$$

Optimal intertemporal choice yields first-order condition

$$MRS \equiv \frac{u'(C_1)}{\beta u'(C_2^+)} = 1 + r$$

where $K_3 = 0$ without bequest motive. ($K_3 = 0$ implies $K_2 + I_2 = 0$ and thus $S_2 = I_2 = -K_2$.) In dubio, assume $K_3 = 0$, i.e. $C_2 = C_2^+$ in the following.

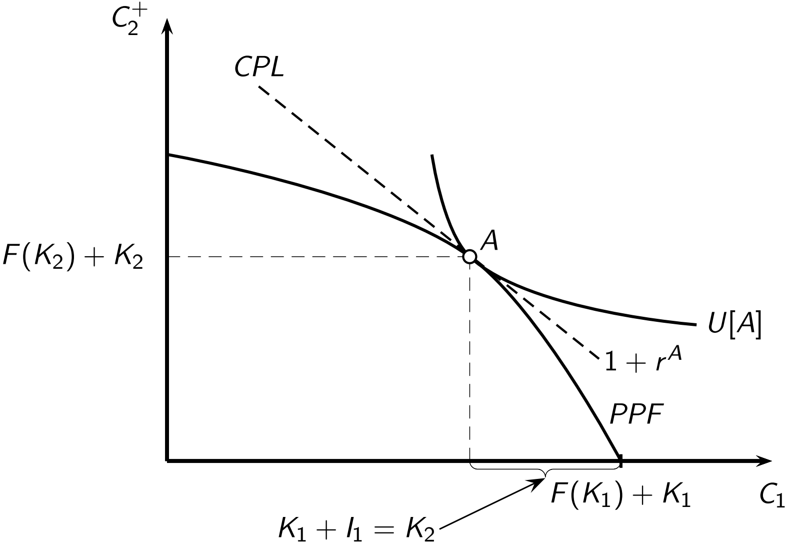


Figure 13: CPL and PPF

Small open economy with capital accumulation and production

Goods market equilibrium

$$C_t + I_t + NX_t = Y_t \tag{24}$$

and intertemporal foreign account (see (11))

$$NX_2 + (1+r)NX_1 = \underbrace{B_3 - (1+r)^2 B_1}_{D}$$

imply

$$\begin{aligned} C_2 + (1+r)C_1 &= Y_2 - I_2 + (1+r)(Y_1 - I_1) - D \\ &= F(K_2) - [K_3 - K_2] + (1+r)[F(K_1) - \\ &\quad (K_2 - K_1)] - D \end{aligned}$$

Hence,

$$\underbrace{C_2 + K_3}_{C_2^+} + (1+r)C_1 = \underbrace{F(K_2) + K_2}_{X_2} + (1+r)\underbrace{[F(K_1) + K_1 - K_2]}_{X_1} - D \quad (25)$$

where $X = (X_1, X_2)$ is a point at the PPF.

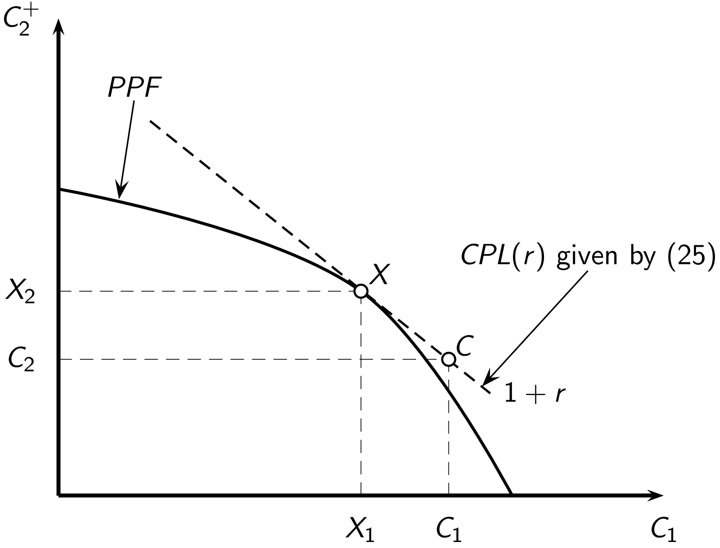


Figure 14: Consumption possibilities line (CPL) under world interest and $D = 0$.

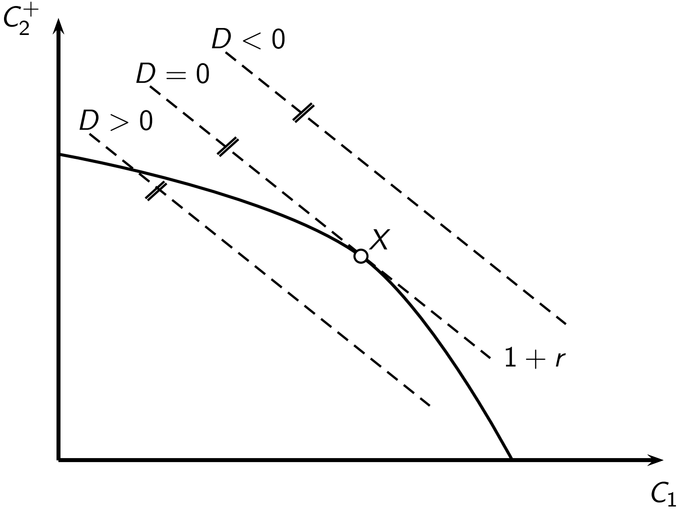


Figure 15: CPL under long-run imbalances ($D \neq 0$)

In the following $D = 0$ (e.g. $B_1 = B_3 = 0$).

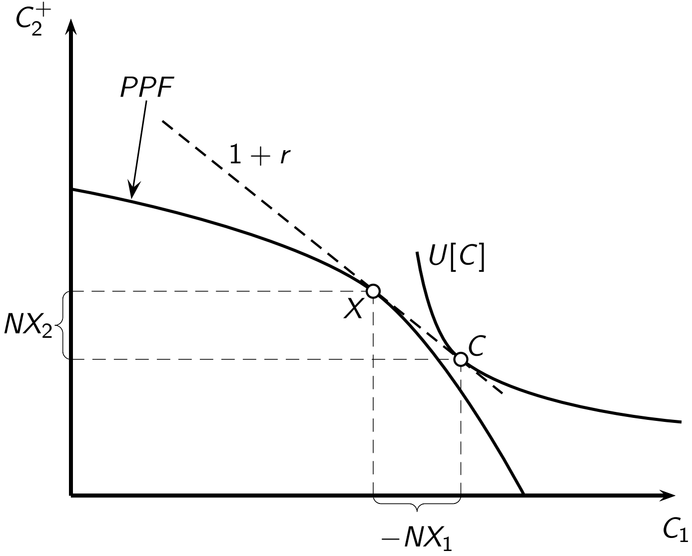


Figure 16: Equilibrium production (X) equilibrium consumption (C), and trade balances (NX)

From autarky to open economy equilibrium

Case $r < r^A$**:**

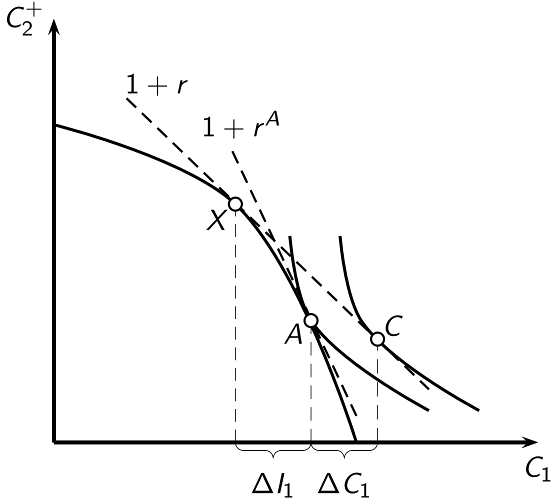


Figure 17: Double gains from intertemporal trade

In addition to the picture for the endowment economy: Production structure shifts from A to X by higher investments ΔI_1.

Increase in current consumption by ΔC_1. Current account deficit $-NX_1 = \Delta I_1 + \Delta C_1$ paid back by increased future production ($+$ possibly lower consumption).

Case $r > r^A$:

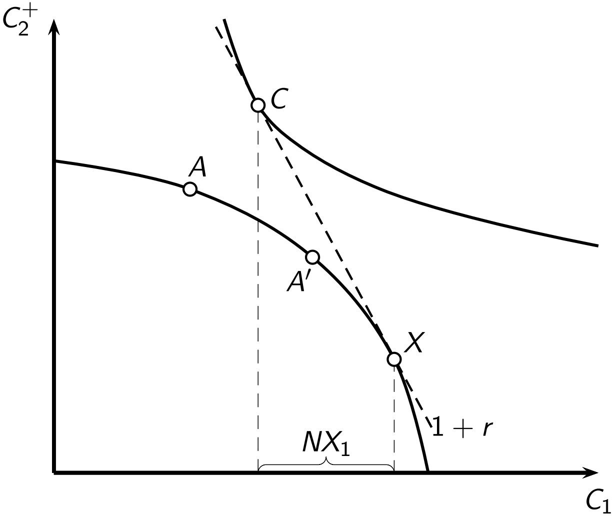


Figure 18: A net exporting country

Production shifts in favor of current output by decreasing investment $\Delta I_1 < 0$. Additional output allows net exports.

Net exports today allow higher future consumption $\Delta C_2 > 0$ by future imports ($NX_2 = -(1+r)NX_1$). Present consumption C_1 may shrink (A') or increase (A) depending on the relative strength of income and substitution effect (plus output shift).

Adding government consumption

With government consumption, period utility has the following additive form: $u(C) + v(G)$. The budget constraint in the two period model is

$$C_1 + \frac{C_2}{1+r} = Y_1 - T_1 - I_1 + \frac{Y_2 - T_2 - I_2}{1+r},$$

where T_t denotes taxes and $Y_t - T_t$ is 'disposable' income of the private sector in period t.

Goods market equilibrium in period t:

$$C_t + I_t + G_t + NX_t = Y_t \tag{26}$$

Current account balance (recall (9),(10)):

$$\begin{aligned} CA_t &= NX_t + rB_t \\ &= Y_t + rB_t - C_t - G_t - I_t \\ &= \underbrace{Y_t + rB_t - C_t - T_t}_{S_t^P \text{ private savings}} + \underbrace{T_t - G_t}_{\text{public savings}} - I_t \end{aligned}$$

With a balanced budget $T_t = G_t$ of the public sector, private savings are equal to total savings ($S_t^P = S_t$) and

$$CA_t = \underbrace{S_t^P + T_t - G_t}_{S_t \text{ total savings}} - I_t \quad (27)$$

$$B_{t+1} = B_t + S_t - I_t \quad (28)$$

Impact of G in small open economy

Increase in $G_1(G_2)$ shifts transformation curve (PPF) for private sector leftward (downward).

In the following illustration (with a balanced budget of the government: $T_t = G_t$):

Initial situation:	$G_1 = G_2 = 0$ and $NX_1 = NX_2 = 0$
Shock 1:	$G_1 \uparrow$
Shock 2:	$G_2 \uparrow$

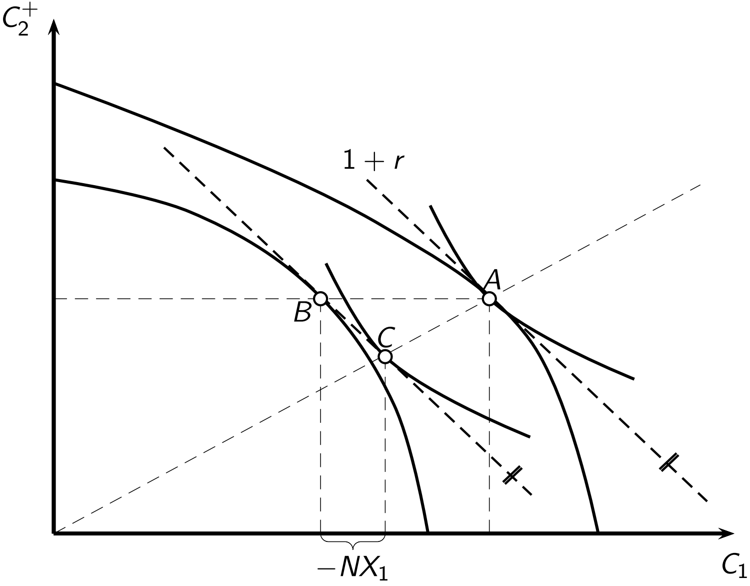


Figure 19: Impact of G_1

Impact of G_1:

- Private feasible output shifts from A to B.
- Would decrease C_1 by the full amount of $G_1 = \overline{BA}$ leaving C_2 unaffected
- Individuals prefer C by borrowing from abroad.

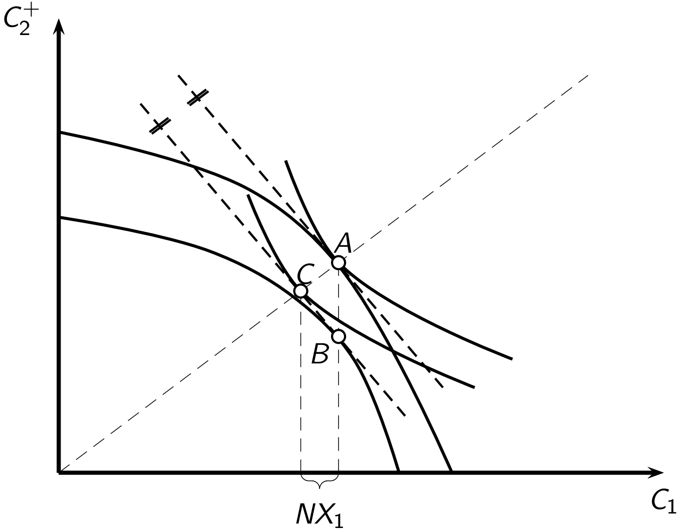


Figure 20: Impact of anticipated G_2 increase

Individuals “hedge” against tax G_2 by lending to *Foreign* in Period 1.

Investment, savings and world interest rate in international equilibrium

The investment function

Production function:

$$Y_t = A_t F(K_t)$$

A_t: Productivity parameter

Accumulation equation:

$$K_2 = K_1 + I_1$$

In the following, we consider a 2-period model with $B_1 = B_3 = 0$, $K_3 = 0$ and $G_t = T_t = 0$.

Condition for optimal capital input under perfect competition:

$$r = A_2 F'(K_1 + I_1) \tag{29}$$

(29) defines investment curve

$$I_1 = I(r/A_2),\ I^{'} < 0$$

The negative slope follows from $F'' < 0$.

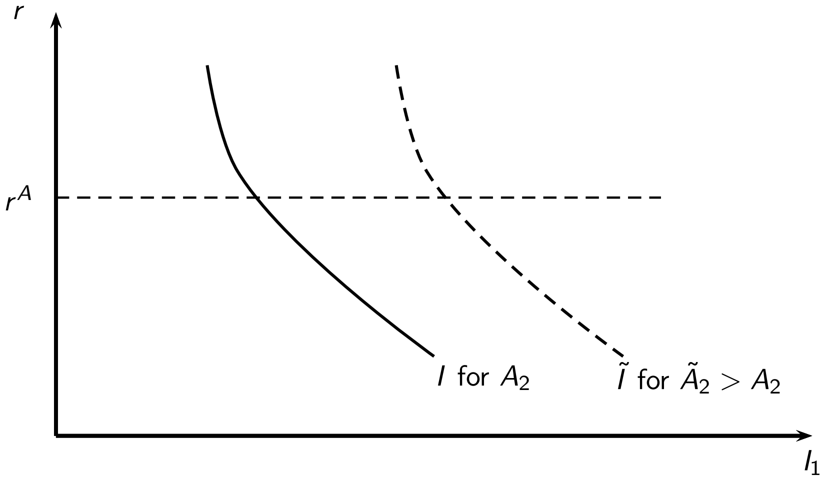


Figure 21: Investment curve and productivity shifts

Shifts in A_1 have no effect on investment since K_1 is already fixed from past decisions.

The saving function

Reconsidering the endowment economy: From the endowment economy we know that $B_1 = B_3 = 0$ implies $S_1 = Y_1 - C_1 = NX_1(r)$.

Furthermore, we can note that, $dS_1/dr = dNX_1/dr = -dC_1/dr$.

To determine the impact of interest rate r on savings (or, equivalently, NX_1), we can first look at the intertemporal Euler equation $u'(C_1) = (1+r)\beta u'(C_2)$.

Substituting the budget constraint $C_2 = (1+r)(Y_1 - C_1) + Y_2$ gives

$$u'(C_1) = (1+r)\beta u'\left((1+r)(Y_1 - C_1) + Y_2\right). \tag{30}$$

Implicitly differentiating (30) with respect to r gives

$$\frac{dC_1}{dr} = \frac{\beta u'(C_2) + \beta(1+r)u''(C_2)(Y_1 - C_1)}{u''(C_1) + \beta(1+r)^2 u''(C_2)}. \tag{31}$$

Noting $u' > 0$, $u'' < 0$, it is immediate that $dNX_1(r)/dr = -dC_1/dr > 0$ if $C_1 > Y_1$ (or, equivalently, $NX_1 < 0$).

However, $dNX_1(r)/dr = -dC_1/dr < 0$ cannot be ruled out if $Y_1 > C_1$ (or, equivalently, $NX_1 > 0$) – see Figure 10.

Consumption in a model with capital accumulation and production

Substituting the budget constraint

$$C_2 = (1+r)[A_1F(K_1) - C_1 - I_1] + A_2F(K_1+I_1) + K_1 + I_1$$

for C_2 in the Euler equation $u'(C_1) = (1+r)\beta u'(C_2)$, gives

$$u'(C_1) = (1+r)\beta u'\left\{(1+r)[A_1F(K_1) - C_1 - I_1] + A_2F(K_1+I_1) + K_1 + I_1\right\}. \qquad (32)$$

Implicitly differentiating with respect to r yields

$$\frac{dC_1}{dr} = \frac{\beta u'(C_2) + \beta(1+r)u''(C_2)\left[A_1F(K_1) - C_1 - I_1\right]}{u''(C_1) + \beta(1+r)^2u''(C_2)} + \frac{\beta(1+r)u''(C_2)\left\{A_2F'(K_1+I_1) - r\right\}\partial I/\partial r}{u''(C_1) + \beta(1+r)^2u''(C_2)}.$$

Accounting for $A_2F'(K_1 + I_1) = r$ further implies

$$\frac{dC_1}{dr} = \frac{\beta u'(C_2) + \beta(1+r)u''(C_2)\left[A_1F(K_1) - C_1 - I_1\right]}{u''(C_1) + \beta(1+r)^2 u''(C_2)}. \tag{33}$$

Hence, the derivative in (33) is precisely the same as the derivative in (31), but with $Y_1 - C_1$ replaced by the date 1 current account for an investment economy with $B_1 = 0$: $A_1F(K_1) - C_1 - I_1$.

That means that, given current account balances, the slope of the saving schedule is the same as for the endowment economy!

An intuition for this result

The symmetry in the reaction of savings to interest rate adjustments in the endowment and the investment economy is a consequence of the *envelope theorem*.

The first-order condition for profit-maximizing investment ensures that a small deviation from optimum investment does not alter the present value of national output, evaluated at the world interest rate.

Consequently, at the margin, the investment adjustment $\partial I_1/\partial r$ has no effect on net lifetime resources, and hence no effect on consumption response.

From consumption to saving

As noted above, savings in period 1 are given by $S_1 = Y_1 - C_1$ or, equivalently, $S_1 = A_1 F(K_1) - C_1$. Hence, we can write savings as function of r, A_1, A_2 and β:

$$S_1 = S(r, A_1, A_2, \beta),$$

with $\partial S_1 / \partial r > 0$ in the regular (*non-perverse*) case.

Saving curve and productivity shift

An increase of A_t has analogous effects to an increase of Y_t in endowment economy.

- According to slide 54 a rise in Y_1 shifts the S-curve to the right. A rise in Y_2 shifts the S-curve to the left.
- Rising impatience (a fall in β) also shifts the saving curve to the left.

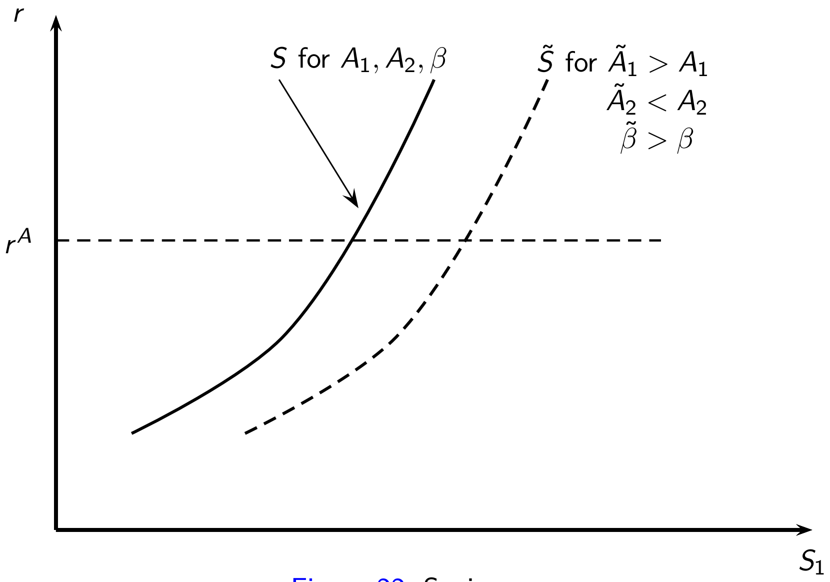


Figure 22: Saving curve

Investment, savings and current account

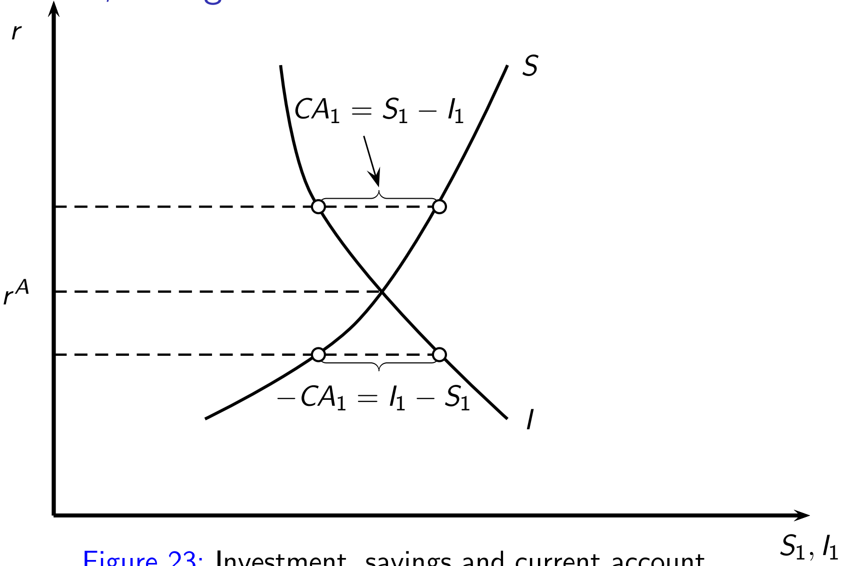


Figure 23: Investment, savings and current account

International equilibrium in a two-region world ("Metzler Diagrams")

World equilibrium requires

$$CA_1 + CA_1^* = 0$$

i.e.

$$S_1 - I_1 = -(S_1^* - I_1^*)$$

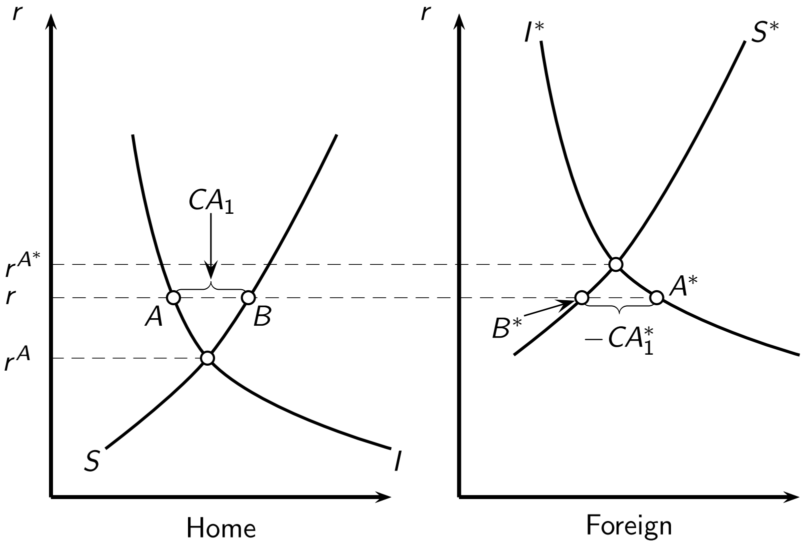


Figure 24: World equilibrium interest rate $r^A < r < r^{A*}$

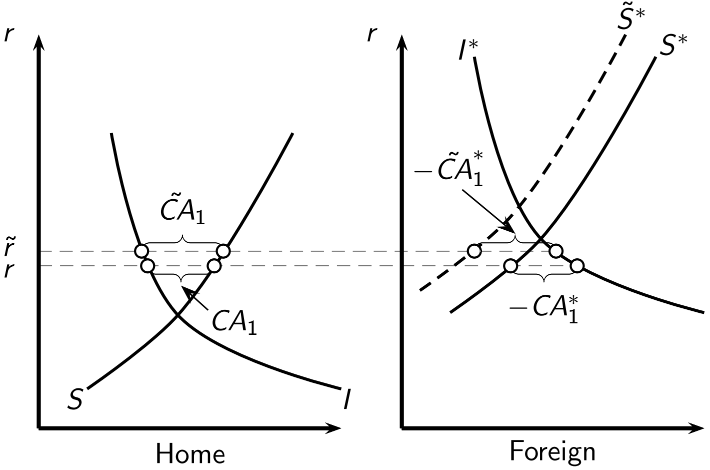


Figure 25: Impact of rising impatience in *Foreign* ($\beta^* \downarrow$)

World interest rate rises and current account CA_1 from *Home* to *Foreign* increases. Investment decreases in both regions.

Impact of positive productivity shock in *Foreign*

Consider a productivity shock of the form $A_2^* \uparrow$

World interest rate rises

In *Home*	Investment falls Saving and CA_1-surplus rise
In *Foreign*	Investment reaction ambiguous CA_1^*-deficit rises

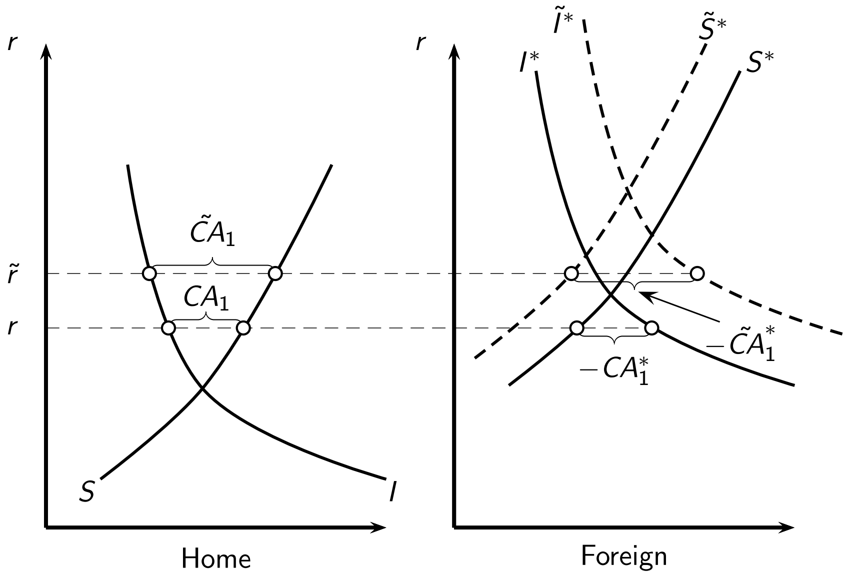


Figure 26: Impact of positive productivity shock in Foreign

Stability of international equilibrium and Marshall-Lerner condition

The world equilibrium condition $CA_t + CA_t^* = 0$ is

$$S_t(r) + S_t^*(r) = I_t(r) + I_t^*(r) \quad (34)$$

(Use $CA_t = S_t - I_t$)

As addressed in Figure 10, the saving curve may be backward bending, so that multiple equilibria and unstable equilibria cannot be excluded.

(Walrasian) stability condition: A market is stable in the Walrasian sense if a small increase in the price of the good traded there causes excess supply, while a small decrease causes excess demand.

The stability condition defining Walrasian stability in the market for world savings is that a small rise in r should lead to an excess supply of savings:

$$\frac{d\left[S_1(r) + S_1^*(r)\right]}{dr} > \frac{d\left[I_1(r) + I_1^*(r)\right]}{dr} \tag{35}$$

Stability guarantees that market forces tend to eliminate imbalances resulting from small disturbances of international equilibrium.

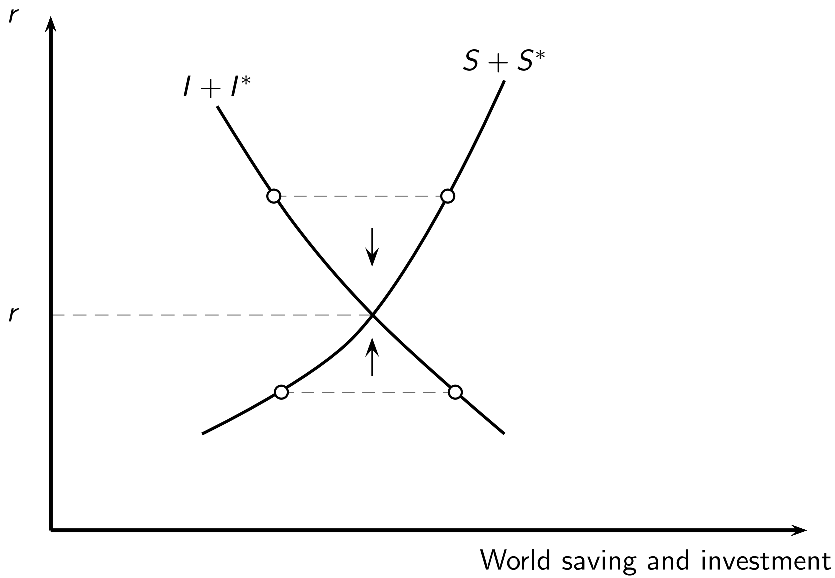


Figure 27: Savings and Investment

For $B_1 = B_3 = 0$ national accounting identities imply

$$\begin{aligned} NX_1 &= CA_1 = S_1 - I_1 \\ NX_1^* &= CA_1^* = S_1^* - I_1^* \end{aligned}$$

Moreover (see (11)),

$$NX_1^* + \frac{NX_2^*}{1+r} = 0$$

Using this in international equilibrium condition (34), we get

$$S_1 - I_1 + S_1^* - I_1^* = NX_1 - \frac{NX_2^*}{1+r}$$

Thus (35) is equivalent to

$$\frac{d\left[NX_1(r) - \frac{NX_2^*(r)}{1+r}\right]}{dr} > 0 \tag{36}$$

$$
\begin{aligned}
\frac{d\left[NX_1(r) - \dfrac{NX_2^*(r)}{1+r}\right]}{dr} &= NX_1' - \frac{NX_2^{*'}(1+r) - NX_2^*}{(1+r)^2} \\
&= \frac{NX_2^*}{(1+r)^2}\left[\frac{(1+r)NX_1'}{NX_1}\frac{NX_1(1+r)}{NX_2^*}\right. \\
&\qquad \left. - \frac{NX_2^{*'}(1+r)}{NX_2^*} + 1\right]
\end{aligned}
$$

In equilibrium $NX_1(1+r) = -NX_2 = NX_2^*$. Thus the square bracket is negative (positive) if

$$\underbrace{-\frac{(1+r)NX_1'}{NX_1}}_{\eta} + \underbrace{\frac{(1+r)NX_2^{*'}}{NX_2^*}}_{\eta^*} > (<)\, 1 \tag{37}$$

If Home is net importer today ($NX_1 < 0$), then $NX_2^* < 0$ and stability condition (35) is equivalent to

$$\eta + \eta^* > 1. \tag{38}$$

Interpretation ($NX_1 < 0$)

η is the (absolute value of) negative import elasticity of Home with respect to price $1 + r$ of current consumption. η^* is the (positive) elasticity of Foreign's future imports. (38) is the intertemporal analogue to the so-called *Marshall-Lerner* condition.

Remark

When *Home* happens to be the exporter in period 1, rather than the importer, (38) still characterizes the Walras-stable case, but with import elasticities defined so that Home's and Foreign's role are interchanged.